Implications of Uncertainty for Optimal Policies

Todd Lensman
Cornell

Maxim Troshkin
Cornell

December, 2018

This project is about:

Mutual implications between:

- Optimal dynamic policy

  (friction-constrained, information or/and no-commitment)

- Broader view of uncertainty

  (Knightian/model/belief uncertainty and risk, aversion to both)

Motivation 1: assumption of uncertainty

Motivation 1: assumption of uncertainty

Recent empirical evidence:

- Pre-tax income distributions change significantly, often
  (e.g. Piketty, Rees-Jones, Saez, Taubinsky, Zuckman, ..)
- People uncertain enough to “leave money on the table”
  (e.g. Aghion, Akcigit, Chetty, Gruber, Lequien, Stantcheva, ..)

Motivation 1: assumption of uncertainty

Recent empirical evidence:

- Pre-tax income distributions change significantly, often
  (e.g. Piketty, Rees-Jones, Saez, Taubinsky, Zuckman, ..)
- People uncertain enough to "leave money on the table"
  (e.g. Aghion, Akcigit, Chetty, Gruber, Lequien, Stantcheva, ..)

Uncertainty in macro / finance:

- Significant explanatory power for economic aggregates
  (e.g. Bianchi, Borovička, Epstein, Hansen, Ilut, Sargent, Schneider, ..)

Motivation 1: assumption of uncertainty

Recent empirical evidence:

- Pre-tax income distributions change significantly, often
  (e.g. Piketty, Rees-Jones, Saez, Taubinsky, Zuckman, ..)
- People uncertain enough to "leave money on the table"
  (e.g. Aghion, Akcigit, Chetty, Gruber, Lequien, Stantcheva, ..)

Uncertainty in macro / finance:

- Significant explanatory power for economic aggregates
  (e.g. Bianchi, Borovička, Epstein, Hansen, Ilut, Sargent, Schneider, ..)

Implications for optimal policies?

- Robust to imperfect knowledge of data-generating process?

Motivation 2: implications of uncertainty

Motivation 2: implications of uncertainty

Optimal policies with certainty about data-generating process:

- once-and-forever (full re-optimize after surprise)
- history-dependent, complex
- complete

Motivation 2: implications of uncertainty

Optimal policies with certainty about data-generating process:

- once-and-forever (full re-optimize after surprise)
- history-dependent, complex
- complete

Commonly-observed policies:

- reformed periodically (especially taxes)
- often do not dependent on full history
- at least somewhat incomplete

Motivation 2: implications of uncertainty

Optimal policies with certainty about data-generating process:

- once-and-forever (full re-optimize after surprise)
- history-dependent, complex
- complete

Commonly-observed policies:

- reformed periodically (especially taxes)
- often do not dependent on full history
- at least somewhat incomplete

Can be optimal?

- Show they can under uncertainty

Results:

Results:

- **Periodically reformed** policies are optimal
  mechanism: uncertainty $\approx$ endogenous no-commitment

Results:

- **Periodically reformed** policies are optimal
  mechanism: uncertainty $\approx$ endogenous no-commitment
  - even with full commitment, information symmetry
  - extends to private beliefs, private skills, exogenous no-commitment

Results:

- **Periodically reformed** policies are optimal
  mechanism: uncertainty $\approx$ endogenous no-commitment
  - even with full commitment, information symmetry
  - extends to private beliefs, private skills, exogenous no-commitment

- **Loss of history** dependence
  mechanism: promise-keeping slack after reform

Results:

- **Periodically reformed** policies are optimal
  mechanism: uncertainty $\approx$ endogenous no-commitment
  - even with full commitment, information symmetry
  - extends to private beliefs, private skills, exogenous no-commitment

- **Loss of history** dependence
  mechanism: promise-keeping slack after reform
  - simplified (**incomplete**) policies more generally
  - if $T < \infty$: no full backward induction for promise utility
  - if linear policies: generically not optimal

Results:

- **Periodically reformed** policies are optimal
  mechanism: uncertainty $\approx$ endogenous no-commitment
  - even with full commitment, information symmetry
  - extends to private beliefs, private skills, exogenous no-commitment

- **Loss of history** dependence
  mechanism: promise-keeping slack after reform
  - simplified (**incomplete**) policies more generally
  - if $T < \infty$: no full backward induction for promise utility
  - if linear policies: generically not optimal

- **Meaningful role for macro interventions**
  mechanism: uncertainty $+$ private info $\Rightarrow$ CE not efficient

Results:

- **Periodically reformed** policies are optimal
  mechanism: uncertainty $\approx$ endogenous no-commitment
  - even with full commitment, information symmetry
  - extends to private beliefs, private skills, exogenous no-commitment

- **Loss of history** dependence
  mechanism: promise-keeping slack after reform
  - simplified (**incomplete**) policies more generally
  - if $T < \infty$: no full backward induction for promise utility
  - if linear policies: generically not optimal

- **Meaningful role for macro interventions**
  mechanism: uncertainty + private info $\Rightarrow$ CE not efficient
  - gov't policies not simply crowding out private insurance
    (contrast: macro policies in the presence of moral hazard)

Uncertainty as friction: baseline setup

Baseline setup

(finite case, infinities in paper)

Baseline setup

(finite case, infinities in paper)

- Time: $t = 0, \ldots, T$
- Agents: $i = 1, \ldots, N$
- Idiosyncratic shocks $s_{i,t}$: **unknowable** finite stochastic process

Baseline setup

(finite case, infinities in paper)

- Time: $t = 0, \ldots, T$
- Agents: $i = 1, \ldots, N$
- Idiosyncratic shocks $s_{i,t}$: **unknowable** finite stochastic process

  $s_{i,t} \equiv (\theta_{i,t}, \Pi_{i,t+1})$:

  - skills $\theta_{i,t}$: effective labor $z_{i,t} = \theta_{i,t} l_{i,t}$
  - beliefs $\Pi_{i,t+1}$: set of distributions over s^{t+1}

Baseline setup

(finite case, infinities in paper)

- Time: $t = 0, \ldots, T$

- Agents: $i = 1, \ldots, N$

- Idiosyncratic shocks $s_{i,t}$: **unknowable** finite stochastic process

  $$s_{i,t} \equiv (\theta_{i,t}, \Pi_{i,t+1}) :$$

  - skills $\theta_{i,t}$: effective labor $z_{i,t} = \theta_{i,t} l_{i,t}$
  - beliefs $\Pi_{i,t+1}$: set of distributions over s^{t+1}

    (agnostic about updating/learning: for simplicity Π in s)

Baseline setup

(finite case, infinities in paper)

- Time: $t = 0, \ldots, T$

- Agents: $i = 1, \ldots, N$

- Idiosyncratic shocks $s_{i,t}$: **unknowable** finite stochastic process

  $$s_{i,t} \equiv (\theta_{i,t}, \Pi_{i,t+1}) :$$

  - skills $\theta_{i,t}$: effective labor $z_{i,t} = \theta_{i,t} l_{i,t}$
  - beliefs $\Pi_{i,t+1}$: set of distributions over s^{t+1}

    (agnostic about updating/learning: for simplicity Π in s)

- Allocation: $C \equiv \left\{c_t\left(s^t\right), z_t\left(s^t\right), k_{t+1}\left(s^t\right)\right\}_{t=0}^{T}$

Aversion to risk and uncertainty

Assume recursive utility:

$$U_{i,t}\left(C|\,s^t\right) \equiv u\left(c_{i,t}\left(s^t\right), \frac{z_{i,t}\left(s^t\right)}{\theta_{i,t}}\right) + \beta \inf_{\Pi_{i,t+1}} \mathbb{E}_{\pi_{i,t+1}}\left[U_{i,t+1}\left(C|\,s^{t+1}\right)\middle|\, s^t\right]$$

- $\pi_{i,t+1} \in \Pi_{i,t+1}$, $\beta \in (0,1)$, $-u_c, u_l < 0$, $u_{cc}, u_{ll} \leq 0$

Aversion to risk and uncertainty

Assume recursive utility:

$$U_{i,t}\left(C \middle| s^t \right) \equiv u\left(c_{i,t}\left(s^t\right), \frac{z_{i,t}\left(s^t\right)}{\theta_{i,t}} \right) + \beta \inf_{\Pi_{i,t+1}} \mathbb{E}_{\pi_{i,t+1}}\left[U_{i,t+1}\left(C \middle| s^{t+1} \right) \middle| s^t \right]$$

- $\pi_{i,t+1} \in \Pi_{i,t+1}$, $\beta \in (0,1)$, $-u_c, u_l < 0$, $u_{cc}, u_{ll} \leq 0$
- Axiomatization, recursive representation: Epstein-Schneider(2003)

Aversion to risk and uncertainty

Assume recursive utility:

$$U_{i,t}\left(C \mid s^t\right) \equiv u\left(c_{i,t}\left(s^t\right), \frac{z_{i,t}\left(s^t\right)}{\theta_{i,t}}\right) + \beta \inf_{\Pi_{i,t+1}} \mathbb{E}_{\pi_{i,t+1}}\left[U_{i,t+1}\left(C \mid s^{t+1}\right) \Big| s^t \right]$$

- $\pi_{i,t+1} \in \Pi_{i,t+1}$, $\beta \in (0,1)$, $-u_c, u_l < 0$, $u_{cc}, u_{ll} \leq 0$
- Axiomatization, recursive representation: Epstein-Schneider(2003)

Results more general:

- dynamic Uncertainty Averse Preferences
  (Cerreia-Vioglio, Maccheroni, Marinacci, Montrucchio 2011)
- ..and nested representations
  (e.g. Multiplier / Model Uncertainty, Hansen-Sargent 2001)
  (e.g. Variational, Maccheroni, Marinacci, Rustichini 2006)
  (e.g. Smooth Ambiguity, Klibanoff, Marinacci, Mukerji 2005)

Government's problem

(start with symmetric information, full commitment)

Government's problem

(start with symmetric information, full commitment)

- C^* is *efficient* given Pareto weights η_i if

$$C^*(s_0) \in \arg\max_C \sum_i U_{i,0}\left(C \mid s_0\right)\eta_i$$

s.t. non-negativity and feasibility:

$$\sum_i c_{i,t}\left(s^t\right) + K_{t+1}\left(s^t\right) \leq f\left(K_t\left(s^{t-1}\right), Z_t\left(s^t\right)\right), \ \ \forall t, s^t \geq s^0$$

Government's problem

(start with symmetric information, full commitment)

- C^* is *efficient* given Pareto weights η_i if

$$C^*(s_0) \in \arg\max_C \sum_i U_{i,0}(C|\, s_0)\, \eta_i$$

s.t. non-negativity and feasibility:

$$\sum_i c_{i,t}\left(s^t\right) + K_{t+1}\left(s^t\right) \leq f\left(K_t\left(s^{t-1}\right), Z_t\left(s^t\right)\right), \ \ \forall t, s^t \geq s^0$$

- ex-post feasibility reflects (heterogeneous) uncertainty

Government's problem

(start with symmetric information, full commitment)

- C^* is *efficient* given Pareto weights η_i if

$$C^*(s_0) \in \arg\max_{C} \sum_i U_{i,0}(C|\, s_0)\, \eta_i$$

s.t. non-negativity and feasibility:

$$\sum_i c_{i,t}\left(s^t\right) + K_{t+1}\left(s^t\right) \leq f\left(K_t\left(s^{t-1}\right), Z_t\left(s^t\right)\right), \quad \forall t, s^t \geq s^0$$

- ex-post feasibility reflects (heterogeneous) uncertainty
- government knows no more than agents

Government's problem

(start with symmetric information, full commitment)

- C^* is *efficient* given Pareto weights η_i if

$$C^*(s_0) \in \arg\max_{C} \sum_i U_{i,0}(C|\, s_0)\, \eta_i$$

s.t. non-negativity and feasibility:

$$\sum_i c_{i,t}(s^t) + K_{t+1}(s^t) \leq f\left(K_t\left(s^{t-1}\right), Z_t\left(s^t\right)\right), \;\; \forall t, s^t \geq s^0$$

- ex-post feasibility reflects (heterogeneous) uncertainty
- government knows no more than agents

- C^*: once-and-forever, typically history dependent, complex

Periodic reforms

Condition on beliefs: Sufficient overlap

..to agree on a feasible path

Condition on beliefs: Sufficient overlap

..to agree on a feasible path

- **Obvious example**: economy's "worst" path
  - skills: $\theta_{i,t} = \underline{\theta}$
  - beliefs: $\underline{\Pi}_{t+1} \equiv \{\underline{\pi}_{t+1}\}$, unit weight on $\theta_{i,t+1} = \underline{\theta}$

Condition on beliefs: Sufficient overlap

..to agree on a feasible path

- **Obvious example**: economy's "worst" path
  - skills: $\theta_{i,t} = \underline{\theta}$
  - beliefs: $\underline{\Pi}_{t+1} \equiv \{\underline{\pi}_{t+1}\}$, unit weight on $\theta_{i,t+1} = \underline{\theta}$

- **Assumption 1**: For any belief $\pi_{i,t+1}$, there is $\pi'_{i,t+1}$:
  - with same marginal distribution of θ (conditional on s^t)
  - but under π' marginal of future Π places unit weight on $\underline{\Pi}$

Condition on beliefs: Sufficient overlap

..to agree on a feasible path

- **Obvious example**: economy's "worst" path
  - skills: $\theta_{i,t} = \underline{\theta}$
  - beliefs: $\underline{\Pi}_{t+1} \equiv \{\underline{\pi}_{t+1}\}$, unit weight on $\theta_{i,t+1} = \underline{\theta}$

- **Assumption 1**: For any belief $\pi_{i,t+1}$, there is $\pi'_{i,t+1}$:
  - with same marginal distribution of θ (conditional on s^t)
  - but under π' marginal of future Π places unit weight on $\underline{\Pi}$

- Relax significantly: ..weight on intervals instead of worst

Condition on beliefs: Sufficient overlap

..to agree on a feasible path

- **Obvious example**: economy's "worst" path
  - skills: $\theta_{i,t} = \underline{\theta}$
  - beliefs: $\underline{\Pi}_{t+1} \equiv \{\underline{\pi}_{t+1}\}$, unit weight on $\theta_{i,t+1} = \underline{\theta}$

- **Assumption 1**: For any belief $\pi_{i,t+1}$, there is $\pi'_{i,t+1}$:
  - with same marginal distribution of θ (conditional on s^t)
  - but under π' marginal of future Π places unit weight on $\underline{\Pi}$

- Relax significantly: ..weight on intervals instead of worst
- Any (heterogeneous) marginals of θ allowed

Condition on beliefs: Sufficient overlap

..to agree on a feasible path

- **Obvious example**: economy's "worst" path
  - skills: $\theta_{i,t} = \underline{\theta}$
  - beliefs: $\underline{\Pi}_{t+1} \equiv \{\underline{\pi}_{t+1}\}$, unit weight on $\theta_{i,t+1} = \underline{\theta}$
- **Assumption 1**: For any belief $\pi_{i,t+1}$, there is $\pi'_{i,t+1}$:
  - with same marginal distribution of θ (conditional on s^t)
  - but under π' marginal of future Π places unit weight on $\underline{\Pi}$
- Relax significantly: ..weight on intervals instead of worst
- Any (heterogeneous) marginals of θ allowed
- DGP not required to place weight on worst path

Result 1: Periodic reforms

Result 1: Periodic reforms

Proposition: Given efficient C^*, there is sequence $\left\{C^t\right\}_{t=0}^{T}$, where $C^t = \left\{c_\tau^t, z_\tau^t, k_{\tau+1}^t\right\}_{\tau=t}^{t+1}$ are incomplete and

$$U_{i,0}\left(C^0 \middle| s^0\right) = U_{i,0}\left(C^* \middle| s^0\right) \quad \forall i,$$

$$U_{i,0}\left(C_0^0, \left(C_t^1\right)_{t=1}^{T} \middle| s^0\right) \geq U_{i,0}\left(C^0 \middle| s^0\right) \quad \forall i,$$

$$U_{i,1}\left(C_1^1, \left(C_t^2\right)_{t=2}^{T} \middle| s^1\right) \geq U_{i,1}\left(C^1 \middle| s^1\right) \quad \forall i,$$

Result 1: Periodic reforms

Proposition: Given efficient C^*, there is sequence $\{C^t\}_{t=0}^{T}$, where $C^t = \{c_\tau^t, z_\tau^t, k_{\tau+1}^t\}_{\tau=t}^{t+1}$ are incomplete and

$$
\begin{aligned}
U_{i,0}\left(C^0 \middle| s^0\right) &= U_{i,0}\left(C^* \middle| s^0\right) \quad \forall i, \\
U_{i,0}\left(C_0^0, \left(C_t^1\right)_{t=1}^{T} \middle| s^0\right) &\geq U_{i,0}\left(C^0 \middle| s^0\right) \quad \forall i, \\
U_{i,1}\left(C_1^1, \left(C_t^2\right)_{t=2}^{T} \middle| s^1\right) &\geq U_{i,1}\left(C^1 \middle| s^1\right) \quad \forall i,
\end{aligned}
$$

Mechanism:

- uncertainty aversion & sufficient belief overlap $\Rightarrow$ need only t & worst-case $t+1$

Result 1: Periodic reforms

Proposition: Given efficient C^*, there is sequence $\left\{C^t\right\}_{t=0}^T$, where $C^t = \left\{c_\tau^t, z_\tau^t, k_{\tau+1}^t\right\}_{\tau=t}^{t+1}$ are incomplete and

$$U_{i,0}\left(C^0 \middle| s^0\right) = U_{i,0}\left(C^* \middle| s^0\right) \quad \forall i,$$

$$U_{i,0}\left(C_0^0, \left(C_t^1\right)_{t=1}^T \middle| s^0\right) \geq U_{i,0}\left(C^0 \middle| s^0\right) \quad \forall i,$$

$$U_{i,1}\left(C_1^1, \left(C_t^2\right)_{t=2}^T \middle| s^1\right) \geq U_{i,1}\left(C^1 \middle| s^1\right) \quad \forall i,$$

Mechanism:

- uncertainty aversion & sufficient belief overlap $\Rightarrow$ need only t & worst-case $t+1$
- when worst not realized $\Rightarrow$ reform $t+1$ & worst-case $t+2$...

Result 1: Periodic reforms

Proposition: Given efficient C^*, there is sequence $\left\{C^t\right\}_{t=0}^{T}$, where $C^t = \left\{c_\tau^t, z_\tau^t, k_{\tau+1}^t\right\}_{\tau=t}^{t+1}$ are incomplete and

$$U_{i,0}\left(C^0 \middle| s^0\right) = U_{i,0}\left(C^* \middle| s^0\right) \quad \forall i,$$
$$U_{i,0}\left(C_0^0, \left(C_t^1\right)_{t=1}^{T} \middle| s^0\right) \geq U_{i,0}\left(C^0 \middle| s^0\right) \quad \forall i,$$
$$U_{i,1}\left(C_1^1, \left(C_t^2\right)_{t=2}^{T} \middle| s^1\right) \geq U_{i,1}\left(C^1 \middle| s^1\right) \quad \forall i,$$

Mechanism:

- uncertainty aversion & sufficient belief overlap $\Rightarrow$ need only t & worst-case $t+1$
- when worst not realized $\Rightarrow$ reform $t+1$ & worst-case $t+2$...
- generalization of incomplete contract ideas (e.g. Mukerji 1998, Zhu 2016)

Result 1: Proof by constructing incomplete C^t

Result 1: Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$,

Result 1: Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$, set C_1^0 to worst-case C_1^*

Result 1: Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$, set C_1^0 to worst-case C_1^*
  - i.e. with $\underline{\Pi}_2$

Result 1: Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$, set C_1^0 to worst-case C_1^*
  - i.e. with $\underline{\Pi}_2$
    (C^0 not fully state contingent, depends only on s^0 and θ_1)

Result 1: Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$, set C_1^0 to worst-case C_1^*
  - i.e. with $\underline{\Pi}_2$
    (C^0 not fully state contingent, depends only on s^0 and θ_1)

- At $t = 0$, all agents : $C^* \sim C^0$
  - $\inf_{\Pi_{i,1}} \mathbb{E}_{\pi_{i,1}} \left[U_{i,1} (.) | \, s^0 \right]$ attains if $\pi_{i,1}$ puts all weight on $\underline{\Pi}_2$

Result 1: Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$, set C_1^0 to worst-case C_1^*
  - i.e. with $\underline{\Pi}_2$
    (C^0 not fully state contingent, depends only on s^0 and θ_1)

- At $t = 0$, all agents : $C^* \sim C^0$
  - $\inf_{\Pi_{i,1}} \mathbb{E}_{\pi_{i,1}} \left[U_{i,1} (.) \middle| s^0 \right]$ attains if $\pi_{i,1}$ puts all weight on $\underline{\Pi}_2$
  - sufficient belief overlap $\Rightarrow$ such $\pi_{i,1}$ exist in $\Pi_{i,1}$

Result 1: Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$, set C_1^0 to worst-case C_1^*
  - i.e. with $\underline{\Pi}_2$
    (C^0 not fully state contingent, depends only on s^0 and θ_1)

- At $t = 0$, all agents : $C^* \sim C^0$
  - $\inf_{\Pi_{i,1}} \mathbb{E}_{\pi_{i,1}} \left[U_{i,1} (.) \middle| s^0 \right]$ attains if $\pi_{i,1}$ puts all weight on $\underline{\Pi}_2$
  - sufficient belief overlap $\Rightarrow$ such $\pi_{i,1}$ exist in $\Pi_{i,1}$

- At $t = 1$, if $\underline{\Pi}_2$ not realized: C_1^0 can be improved to C_1^1 & worst-case C_2^1, and so on..

Result 1: Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$, set C_1^0 to worst-case C_1^*
  - i.e. with $\underline{\Pi}_2$
    (C^0 not fully state contingent, depends only on s^0 and θ_1)

- At $t = 0$, all agents : $C^* \sim C^0$
  - $\inf_{\Pi_{i,1}} \mathbb{E}_{\pi_{i,1}} \left[U_{i,1}(.) \middle| s^0 \right]$ attains if $\pi_{i,1}$ puts all weight on $\underline{\Pi}_2$
  - sufficient belief overlap $\Rightarrow$ such $\pi_{i,1}$ exist in $\Pi_{i,1}$

- At $t = 1$, if $\underline{\Pi}_2$ not realized: C_1^0 can be improved to C_1^1 & worst-case C_2^1, and so on..
  - C_1^0 still feasible

Result 1: Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$, set C_1^0 to worst-case C_1^*
  - i.e. with $\underline{\Pi}_2$
    (C^0 not fully state contingent, depends only on s^0 and θ_1)

- At $t = 0$, all agents : $C^* \sim C^0$
  - $\inf_{\Pi_{i,1}} \mathbb{E}_{\pi_{i,1}} \left[U_{i,1} (.) | \, s^0 \right]$ attains if $\pi_{i,1}$ puts all weight on $\underline{\Pi}_2$
  - sufficient belief overlap $\Rightarrow$ such $\pi_{i,1}$ exist in $\Pi_{i,1}$

- At $t = 1$, if $\underline{\Pi}_2$ not realized: C_1^0 can be improved to C_1^1 & worst-case C_2^1, and so on..
  - C_1^0 still feasible
  - ..so acts like *endogenous outside option (fallback)*

Characterize incomplete C^t without knowing C^*?

Characterize incomplete C^t without knowing C^*?

Government's reform problem:

Given C^{t-1}, efficient to reform to

$$C^t\left(s^t, C^{t-1}\right) \in \arg\max_{C^t} \sum_i U_{i,t}\left(C^t \middle| s^t\right) \eta_i$$

Characterize incomplete C^t without knowing C^*?

Government's reform problem:

Given C^{t-1}, efficient to reform to

$$C^t\left(s^t, C^{t-1}\right) \in \arg\max_{C^t} \sum_i U_{i,t}\left(C^t \middle| s^t\right)\eta_i$$

s.t. non-negativity, feasibility $\tau = t, t+1$, $s^\tau \geq s^t$

$$\sum_i c^t_{i,\tau}\left(s^\tau\right) + K^t_{\tau+1}\left(s^\tau\right) \leq f\left(K^{\tau-1}_\tau\left(s^{\tau-1}\right), Z^t_\tau\left(s^\tau\right)\right),$$

Characterize incomplete C^t without knowing C^*?

Government's reform problem:

Given C^{t-1}, efficient to reform to

$$C^t\left(s^t, C^{t-1}\right) \in \arg\max_{C^t} \sum_i U_{i,t}\left(C^t \middle| s^t\right) \eta_i$$

s.t. non-negativity, feasibility $\tau = t, t+1$, $s^\tau \geq s^t$

$$\sum_i c^t_{i,\tau}\left(s^\tau\right) + K^t_{\tau+1}\left(s^\tau\right) \leq f\left(K^{\tau-1}_\tau\left(s^{\tau-1}\right), Z^t_\tau\left(s^\tau\right)\right),$$

and promise-keeping $\forall i$

$$U_{i,t-1}\left(C^{t-1}_{t-1}, \left(C^t_\tau\right)^T_{\tau=t} \middle| s^t\right) \geq U_{i,t-1}\left(C^{t-1} \middle| s^t\right)$$

Discussion

Government's reform problem:

Discussion

Government's reform problem:

- simplified algorithm for constructing optimal allocations

Discussion

Government's reform problem:

- simplified algorithm for constructing optimal allocations
- previous allocation C^{t-1}: fallback option

Discussion

Government's reform problem:

- simplified algorithm for constructing optimal allocations
- previous allocation C^{t-1}: fallback option
- **periodic reforms** $\approx$ form of endogenous lack of commitment (on the part of gov't)

Discussion

Government's reform problem:

- simplified algorithm for constructing optimal allocations
- previous allocation C^{t-1}: fallback option
- **periodic reforms** $\approx$ form of endogenous lack of commitment (on the part of gov't)

Simplified / incomplete C^t:

- limited dependence on future shocks, distributions

Discussion

Government's reform problem:

- simplified algorithm for constructing optimal allocations
- previous allocation C^{t-1}: fallback option
- **periodic reforms** $\approx$ form of endogenous lack of commitment (on the part of gov't)

Simplified / incomplete C^t:

- limited dependence on future shocks, distributions
- **history dependence** only via promise-keeping (conditioning in beliefs only)

Takeaways

Moving away from certainty about DGP:

Takeaways

Moving away from certainty about DGP:

- simplified, more realistic optimal policies
  - reformed periodically, incomplete, not fully history dependent

Takeaways

Moving away from certainty about DGP:

- simplified, more realistic optimal policies
  - reformed periodically, incomplete, not fully history dependent
- simplified computation of optima
  - no full backward induction

Takeaways

Moving away from certainty about DGP:

- simplified, more realistic optimal policies
  - reformed periodically, incomplete, not fully history dependent
- simplified computation of optima
  - no full backward induction
- meaningful role for gov't intervention
  - beyond crowding out private insurance

Takeaways

Moving away from certainty about DGP:

- simplified, more realistic optimal policies
  - reformed periodically, incomplete, not fully history dependent
- simplified computation of optima
  - no full backward induction
- meaningful role for gov't intervention
  - beyond crowding out private insurance
- ..but affine policies generically not optimal

Supplementary Slides

History independence

Result 2: History independence

Proposition: C^t is independent of full history whenever and reform leads to improvement (assume beliefs are Markov)

- i.e. when promise-keeping slack in the gov's reform problem

Result 2: History independence

Proposition: C^t is independent of full history whenever and reform leads to improvement (assume beliefs are Markov)

- i.e. when promise-keeping slack in the gov's reform problem

Example: whenever C^t can be constructed by backward induction

Dynamic consistency?

Preferences *dynamically consistent* in natural sense

Dynamic consistency?

Preferences *dynamically consistent* in natural sense :

- If $C, \tilde{C}$ coincide at t and for all $s^{t+1} \geq s^t$

  $$U_{i,t}\left(C \mid s^{t+1}\right) \leq U_{i,t}\left(\tilde{C} \mid s^{t+1}\right),$$

  then

  $$U_{i,t}\left(C \mid s^{t}\right) \leq U_{i,t}\left(\tilde{C} \mid s^{t}\right)$$

Dynamic consistency?

Preferences *dynamically consistent* in natural sense :

- If $C, \tilde{C}$ coincide at t and for all $s^{t+1} \geq s^t$

$$U_{i,t}\left(C \middle| s^{t+1} \right) \leq U_{i,t}\left(\tilde{C} \middle| s^{t+1} \right),$$

  then

$$U_{i,t}\left(C \middle| s^{t} \right) \leq U_{i,t}\left(\tilde{C} \middle| s^{t} \right)$$

  - immediate from recursive rep. of $U_{i,t}$
  - current beliefs are not allocation dependent

Dynamic consistency?

Preferences *dynamically consistent* in natural sense :

- If $C, \tilde{C}$ coincide at t and for all $s^{t+1} \geq s^t$

$$U_{i,t}\left(C \middle| s^{t+1} \right) \leq U_{i,t}\left(\tilde{C} \middle| s^{t+1} \right),$$

  then

$$U_{i,t}\left(C \middle| s^{t} \right) \leq U_{i,t}\left(\tilde{C} \middle| s^{t} \right)$$

  - immediate from recursive rep. of $U_{i,t}$
  - current beliefs are not allocation dependent
- Same notion as:
  - Epstein-Schneider(2003), Maccheroni, Marinacci, Rustichini(2006), Klibanoff, Marinacci, Mukerji(2005), etc.
- Implies:
  - agents can find ex-ante solution by backward induction (weaker/more policy-relevant, e.g. Hansen-Sargent 2001 multiplier)

Private shocks

Private shocks

Private shocks

- Observed state at beginning of t : $\hat{s}^{t-1} = \left(\hat{\theta}^{t-1}, \hat{\Pi}^{t}\right)$
    - $t-1$ history of skill reports
    - $t-1$ history of belief reports: $\hat{\Pi}_{i,t}$ reported at $t-1$

Private shocks

- Observed state at beginning of t : $\hat{s}^{t-1} = \left(\hat{\theta}^{t-1}, \hat{\Pi}^{t}\right)$
  - $t-1$ history of skill reports
  - $t-1$ history of belief reports: $\hat{\Pi}_{i,t}$ reported at $t-1$

- Reporting strategy: $\sigma_i = \{\sigma_{i,t}\}_{t=0}^{T}$
  - where $\sigma_{i,t}\left(\hat{s}^{t-1}, s_i^t\right) = \left(\hat{\theta}_{i,t}, \hat{\Pi}_{i,t+1}\right)$
  - truth-telling: σ_i^*

Private shocks

- Observed state at beginning of t : $\hat{s}^{t-1} = \left(\hat{\theta}^{t-1}, \hat{\Pi}^{t}\right)$
  - $t-1$ history of skill reports
  - $t-1$ history of belief reports: $\hat{\Pi}_{i,t}$ reported at $t-1$

- Reporting strategy: $\sigma_i = \{\sigma_{i,t}\}_{t=0}^{T}$
  - where $\sigma_{i,t}\left(\hat{s}^{t-1}, s_i^t\right) = \left(\hat{\theta}_{i,t}, \hat{\Pi}_{i,t+1}\right)$
  - truth-telling: σ_i^*

- **Revelation Principle** holds
  - consider only incentive compatible C:

$$U_{i,0}\left(C \mid s_{i,0}\right)\left(\sigma^*\right) \geq U_{i,0}\left(C \mid s_{i,0}\right)\left(\sigma_i, \sigma_{-i}^*\right) \qquad \forall i, \sigma_i, s_{i,0}$$

Condition on beliefs, allocations

..sufficient overlap to agree on a feasible path (as before)

Condition on beliefs, allocations

..sufficient overlap to agree on a feasible path (as before)

- Private info $\Rightarrow$ beliefs about current state matter now

  many "worst" beliefs $\underline{\underline{\Pi}}_{i,t+1}$: all place unit weight on $\theta_{j,t+1} = \underline{\theta}$ and $\underline{\underline{\Pi}}_{j,t+1}$, but any marginals over current θ

Condition on beliefs, allocations

..sufficient overlap to agree on a feasible path (as before)

- Private info $\Rightarrow$ beliefs about current state matter now

  many "worst" beliefs $\underline{\underline{\Pi}}_{i,t+1}$: all place unit weight on $\theta_{j,t+1} = \underline{\theta}$ and $\underline{\underline{\Pi}}_{j,t+1}$, but any marginals over current θ

- **Assumption 1'**: For any belief $\pi_{i,t+1}$, there is $\pi'_{i,t+1}$:
  - with same marginal distribution of θ (conditional on $\hat{s}^{t-1}$, s_i^t)
  - but under π' marginal of future Π places unit weight on $\underline{\underline{\Pi}}$

Condition on beliefs, allocations

..sufficient overlap to agree on a feasible path (as before)

- Private info $\Rightarrow$ beliefs about current state matter now

  many "worst" beliefs $\underline{\underline{\Pi}}_{i,t+1}$: all place unit weight on $\theta_{j,t+1} = \underline{\theta}$ and $\underline{\underline{\Pi}}_{j,t+1}$, but any marginals over current θ

- **Assumption 1'**: For any belief $\pi_{i,t+1}$, there is $\pi'_{i,t+1}$:
  - with same marginal distribution of θ (conditional on $\hat{s}^{t-1}$, s_i^t)
  - but under π' marginal of future Π places unit weight on $\underline{\underline{\Pi}}$

- **Assumption 2**: *weak monotonicity* of allocations

Condition on beliefs, allocations

..sufficient overlap to agree on a feasible path (as before)

- Private info $\Rightarrow$ beliefs about current state matter now

  many "worst" beliefs $\underline{\underline{\Pi}}_{i,t+1}$: all place unit weight on $\theta_{j,t+1} = \underline{\theta}$ and $\underline{\underline{\Pi}}_{j,t+1}$, but any marginals over current θ

- **Assumption 1'**: For any belief $\pi_{i,t+1}$, there is $\pi'_{i,t+1}$:
  - with same marginal distribution of θ (conditional on $\hat{s}^{t-1}$, s_i^t)
  - but under π' marginal of future Π places unit weight on $\underline{\underline{\Pi}}$

- **Assumption 2**: *weak monotonicity* of allocations
  - weakly worse off if all others certain to report $\underline{\theta}$ at t, $t+1$, ..

Condition on beliefs, allocations

..sufficient overlap to agree on a feasible path (as before)

- Private info $\Rightarrow$ beliefs about current state matter now

  many "worst" beliefs $\underline{\underline{\Pi}}_{i,t+1}$: all place unit weight on $\theta_{j,t+1} = \underline{\theta}$ and $\underline{\underline{\Pi}}_{j,t+1}$, but any marginals over current θ

- **Assumption 1'**: For any belief $\pi_{i,t+1}$, there is $\pi'_{i,t+1}$:
  - with same marginal distribution of θ (conditional on $\hat{s}^{t-1}$, s_i^t)
  - but under π' marginal of future Π places unit weight on $\underline{\underline{\Pi}}$

- **Assumption 2**: *weak monotonicity* of allocations
  - weakly worse off if all others certain to report $\underline{\theta}$ at t, $t+1$, ..

(resources-worst coincides with subjective continuation-utility-worst)

Incentive compatibility?

Sufficient belief overlap:

- construct C^0 like with public information
- $C_0^0 = C_0^*$, C_0^1 assumes worst beliefs $\underline{\underline{\Pi}}_{j,2}$ $\forall j$

Incentive compatibility?

Sufficient belief overlap:

- construct C^0 like with public information
- $C_0^0 = C_0^*$, C_0^1 assumes worst beliefs $\underline{\underline{\Pi}}_{j,2}$ $\forall j$

Weak monotonicity in C^*:

- only consider worst path when choosing strategy

Incentive compatibility?

Sufficient belief overlap:

- construct C^0 like with public information
- $C_0^0 = C_0^*$, C_0^1 assumes worst beliefs $\underline{\underline{\Pi}}_{j,2}$ $\forall j$

Weak monotonicity in C^*:

- only consider worst path when choosing strategy

  $\Rightarrow$ C^0 incentive compatible

Reforming

$t = 1$: reform to new allocation C^1 if possible

- s.t. IC, promise-keeping, threat-keeping

Reforming

$t = 1$: reform to new allocation C^1 if possible

- s.t. IC, promise-keeping, threat-keeping
- if constraint set empty: just keep old C^0 (+recommend eq'm)

Reforming

$t = 1$: reform to new allocation C^1 if possible

- s.t. IC, promise-keeping, threat-keeping
- if constraint set empty: just keep old C^0 (+recommend eq'm)

$t = 2$: reform to new allocation C^2 if possible ...

Extension:

Exogenous lack of commitment

Extension:

Exogenous lack of commitment

- each agent has outside option $\underline{U}_{i,t}\left(s_i^t\right)$
- in government's reform problem:
  - new C^t must also satisfy self-enforcement :

$$U_{i,t}\left(C^t \middle| \hat{s}^{t-1}, s_i^t\right)(\sigma^*) \geq \underline{U}_{i,t}\left(s_i^t\right)$$

Inefficiency of competitive equilibria

Decentralization

- Competitive firms, contract one-to-one with agents:
  - buy k_0, employ $z_{i,t}$, produce $f\left(k_{i,t}, z_{i,t}\right)$, return $c_{i,t}$

Decentralization

- Competitive firms, contract one-to-one with agents:
  - buy k_0, employ $z_{i,t}$, produce $f\left(k_{i,t}, z_{i,t}\right)$, return $c_{i,t}$
  - adopt agents' beliefs $\Pi_{i,t+1}$

Decentralization

- Competitive firms, contract one-to-one with agents:
  - buy k_0, employ $z_{i,t}$, produce $f\left(k_{i,t}, z_{i,t}\right)$, return $c_{i,t}$
  - adopt agents' beliefs $\Pi_{i,t+1}$
  - reinterpretation: agents have direct access to f, securities markets

Decentralization

- Competitive firms, contract one-to-one with agents:
  - buy k_0, employ $z_{i,t}$, produce $f\left(k_{i,t}, z_{i,t}\right)$, return $c_{i,t}$
  - adopt agents' beliefs $\Pi_{i,t+1}$
  - reinterpretation: agents have direct access to f, securities markets

- **Start**: *all* Arrow-Debreu securities

Decentralization

- Competitive firms, contract one-to-one with agents:
  - buy k_0, employ $z_{i,t}$, produce $f\left(k_{i,t}, z_{i,t}\right)$, return $c_{i,t}$
  - adopt agents' beliefs $\Pi_{i,t+1}$
  - reinterpretation: agents have direct access to f, securities markets

- **Start**: *all* Arrow-Debreu securities

- **Lemma:** Securities contingent on idiosyncratic reports $\hat{s}_i^t$ not traded in CE

Decentralization

- Competitive firms, contract one-to-one with agents:
  - buy k_0, employ $z_{i,t}$, produce $f\left(k_{i,t}, z_{i,t}\right)$, return $c_{i,t}$
  - adopt agents' beliefs $\Pi_{i,t+1}$
  - reinterpretation: agents have direct access to f, securities markets

- **Start**: *all* Arrow-Debreu securities

- **Lemma:** Securities contingent on idiosyncratic reports $\hat{s}_i^t$ not traded in CE
  - immediate from arbitrage vs. risk-free bonds (e.g. Golosov-Tsyvinski 2007)

Decentralization

- Competitive firms, contract one-to-one with agents:
  - buy k_0, employ $z_{i,t}$, produce $f\left(k_{i,t}, z_{i,t}\right)$, return $c_{i,t}$
  - adopt agents' beliefs $\Pi_{i,t+1}$
  - reinterpretation: agents have direct access to f, securities markets

- **Start**: *all* Arrow-Debreu securities

- **Lemma:** Securities contingent on idiosyncratic reports $\hat{s}_i^t$ not traded in CE
  - immediate from arbitrage vs. risk-free bonds (e.g. Golosov-Tsyvinski 2007)

- **Result 3:** CE may not be efficient

Only risk-free bonds in equilibrium

Lemma. Securities contingent on idiosyncratic reports $\hat{s}_i^t$ are not traded in CE.

- Security $a(\hat{s}_i^t)$ pays if agent i reports $\hat{s}_i^t$
- Suppose $a(\hat{s}_i^t)$ costs strictly less than risk-free bond:
  - i buys ∞ $a(\hat{s}_i^t)$ and sells ∞ risk-free bonds, reports $\hat{s}_i^t$ at t
  - i nets ∞ profit, sellers of $a(\hat{s}_i^t)$ guaranteed to lose $\rightarrow\leftarrow$

$\Rightarrow$ only risk-free bonds traded in CE

Example: CE inefficiency

$N = 2$: $\Pi_{A,1} = \left\{ \underline{\pi}_{A,1}, \bar{\pi}_{A,1} \right\}$, $\Pi_{B,1} = \{ \bar{\pi}_{B,1} \}$

Example: CE inefficiency

$N = 2$: $\Pi_{A,1} = \left\{\underline{\pi}_{A,1}, \bar{\pi}_{A,1}\right\}$, $\Pi_{B,1} = \{\bar{\pi}_{B,1}\}$

- under $\underline{\pi}_{A,1}$, at $t \geq 1$ all agents certain to realize $\underline{\theta}$

Example: CE inefficiency

$N = 2$: $\Pi_{A,1} = \left\{\underline{\pi}_{A,1}, \bar{\pi}_{A,1}\right\}$, $\Pi_{B,1} = \{\bar{\pi}_{B,1}\}$

- under $\underline{\pi}_{A,1}$, at $t \geq 1$ all agents certain to realize $\underline{\theta}$
  - also believes other agent realized $\Pi_{B,1} = \left\{\underline{\pi}_{B,1}, \bar{\pi}_{B,1}\right\}$

Example: CE inefficiency

$N = 2$: $\Pi_{A,1} = \left\{\underline{\pi}_{A,1}, \bar{\pi}_{A,1}\right\}$, $\Pi_{B,1} = \{\bar{\pi}_{B,1}\}$

- under $\underline{\pi}_{A,1}$, at $t \geq 1$ all agents certain to realize $\underline{\theta}$
  - also believes other agent realized $\Pi_{B,1} = \left\{\underline{\pi}_{B,1}, \bar{\pi}_{B,1}\right\}$
- under $\bar{\pi}_{A,1}$, at $t \geq 1$ all agents certain to realize $\bar{\theta}$

Example: CE inefficiency

$N = 2$: $\Pi_{A,1} = \left\{\underline{\pi}_{A,1}, \bar{\pi}_{A,1}\right\}$, $\Pi_{B,1} = \{\bar{\pi}_{B,1}\}$

- under $\underline{\pi}_{A,1}$, at $t \geq 1$ all agents certain to realize $\underline{\theta}$
  - also believes other agent realized $\Pi_{B,1} = \left\{\underline{\pi}_{B,1}, \bar{\pi}_{B,1}\right\}$
- under $\bar{\pi}_{A,1}$, at $t \geq 1$ all agents certain to realize $\bar{\theta}$
  - also believes other agent realized $\Pi_{B,1} = \left\{\underline{\pi}_{B,1}, \bar{\pi}_{B,1}\right\}$

Example: CE inefficiency

$N = 2$: $\Pi_{A,1} = \{\underline{\pi}_{A,1}, \bar{\pi}_{A,1}\}$, $\Pi_{B,1} = \{\bar{\pi}_{B,1}\}$

- under $\underline{\pi}_{A,1}$, at $t \geq 1$ all agents certain to realize $\underline{\theta}$
  - also believes other agent realized $\Pi_{B,1} = \{\underline{\pi}_{B,1}, \bar{\pi}_{B,1}\}$
- under $\bar{\pi}_{A,1}$, at $t \geq 1$ all agents certain to realize $\bar{\theta}$
  - also believes other agent realized $\Pi_{B,1} = \{\underline{\pi}_{B,1}, \bar{\pi}_{B,1}\}$

- $\underline{\pi}_{B,1}, \bar{\pi}_{B,1}$ defined symmetrically

Example: CE inefficiency

Planner:

- would transfer consumption to A when all realize $\underline{\theta}$ at $t = 1$

Example: CE inefficiency

Planner:

- would transfer consumption to A when all realize $\underline{\theta}$ at $t = 1$
- IC satisfied: B does not believe A will realize $\theta_{A,1} = \underline{\theta}$

Example: CE inefficiency

Planner:

- would transfer consumption to A when all realize $\underline{\theta}$ at $t = 1$
- IC satisfied: B does not believe A will realize $\theta_{A,1} = \underline{\theta}$

CE:

- A would have to insure by purchasing risk-free bond
- $\Rightarrow t = 0$ instantaneous utility lower than efficient

Example: CE inefficiency

Planner:

- would transfer consumption to A when all realize $\underline{\theta}$ at $t = 1$
- IC satisfied: B does not believe A will realize $\theta_{A,1} = \underline{\theta}$

CE:

- A would have to insure by purchasing risk-free bond
- $\Rightarrow t = 0$ instantaneous utility lower than efficient

Note: nothing prevents decentralized periodic reforms, history independence, incompleteness

Periodic reforms in equilibrium

- At $t = 0$, agent i solves for fully continent allocation

$$C_i = \left\{ c_{i,t}\left(s^t\right), z_{i,t}\left(s^t\right), k_{i,t+1}\left(s^t\right), b_{i,t+1}\left(s^{t-1}\right) \right\}_{t=0}^{T}$$

  - given risk-free bond prices $\left\{ Q\left(s^t\right) \right\}_{t=0}^{T-1}$

Periodic reforms in equilibrium

- At $t = 0$, agent i solves for fully continent allocation

$$C_i = \left\{ c_{i,t}\left(s^t\right), z_{i,t}\left(s^t\right), k_{i,t+1}\left(s^t\right), b_{i,t+1}\left(s^{t-1}\right) \right\}_{t=0}^{T}$$

  - given risk-free bond prices $\left\{ Q\left(s^t\right) \right\}_{t=0}^{T-1}$

Proposition: For any $C = \left\{ C_i \right\}_{i=1}^{N}$, there exist incomplete allocations $\left\{ C^t \right\}_{t=0}^{T}$ such that

$$U_{i,0}\left(C \middle| s_0 \right) = U_{i,0}\left(C^0 \middle| s_0 \right) \ \forall i, s_0$$

Periodic reforms in equilibrium

- At $t = 0$, agent i solves for fully continent allocation

$$C_i = \left\{ c_{i,t}\left(s^t\right), z_{i,t}\left(s^t\right), k_{i,t+1}\left(s^t\right), b_{i,t+1}\left(s^{t-1}\right) \right\}_{t=0}^{T}$$

  - given risk-free bond prices $\{Q\left(s^t\right)\}_{t=0}^{T-1}$

Proposition: For any $C = \{C_i\}_{i=1}^{N}$, there exist incomplete allocations $\{C^t\}_{t=0}^{T}$ such that

$$U_{i,0}\left(C \mid s_0\right) = U_{i,0}\left(C^0 \mid s_0\right) \; \forall i, s_0$$

- Periodic reforms decentralized: each C^t designed assuming that all agents receive worst beliefs $\underline{\Pi}_{t+2}$ and worst shock $\underline{\theta}$ at $\tau \geq t+2$

Taking simplicity further: Linearity?

Simplified policies

- **Simplified** optimal policies $\Leftarrow$ periodic reforms
    - no need for full backward induction, period-at-a-time
    - no need for full history dependence

Simplified policies

- **Simplified** optimal policies $\Leftarrow$ periodic reforms
  - no need for full backward induction, period-at-a-time
  - no need for full history dependence
- **Affine** optimal policies?
  - **not generically** in the sense of fiscal policies

Simplified policies

- **Simplified** optimal policies $\Leftarrow$ periodic reforms
  - no need for full backward induction, period-at-a-time
  - no need for full history dependence
- **Affine** optimal policies?
  - **not generically** in the sense of fiscal policies
  - unlike in contracting (Carroll 2014, Zhu 2016)

Simplified policies

- **Simplified** optimal policies $\Leftarrow$ periodic reforms
  - no need for full backward induction, period-at-a-time
  - no need for full history dependence
- **Affine** optimal policies?
  - **not generically** in the sense of fiscal policies
  - unlike in contracting (Carroll 2014, Zhu 2016)

Typical example that works ($N < \infty$ agents) :

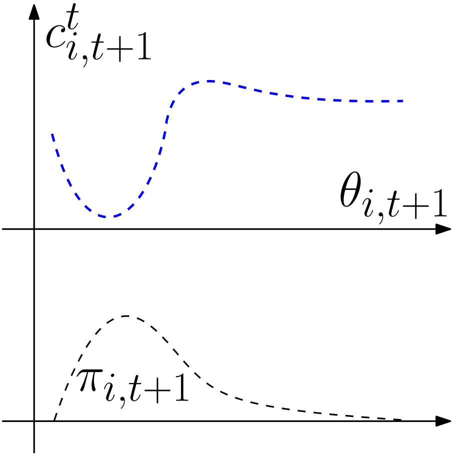
- inelastic labor supply
- agents believe skill shocks independently distributed
  (key results continue to hold)

Affine optimal policies?

Lemma. At any t, any agent i weakly prefers affine $\overline{c}^t_{i,t+1}$ to nonlinear $c^t_{i,t+1}$ (affine in $\theta_{i,t+1}$)

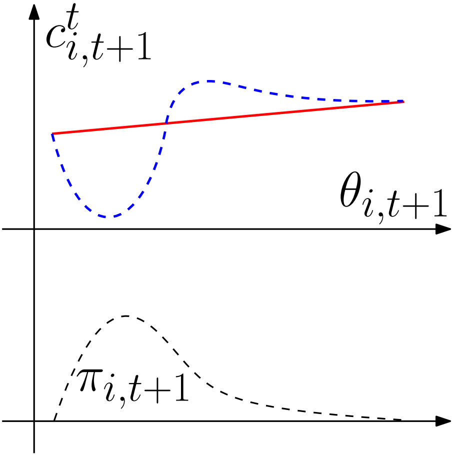
Affine optimal policies?

Lemma. At any t, any agent i weakly prefers affine $\overline{c}^t_{i,t+1}$ to nonlinear $c^t_{i,t+1}$ (affine in $\theta_{i,t+1}$)



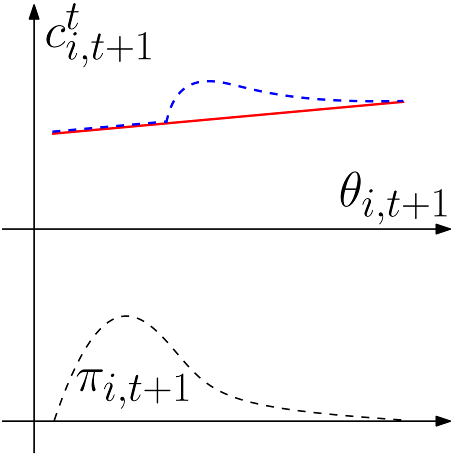
Affine optimal policies?

Lemma. At any t, any agent i weakly prefers affine $\overline{c}^t_{i,t+1}$ to nonlinear $c^t_{i,t+1}$ (affine in $\theta_{i,t+1}$)



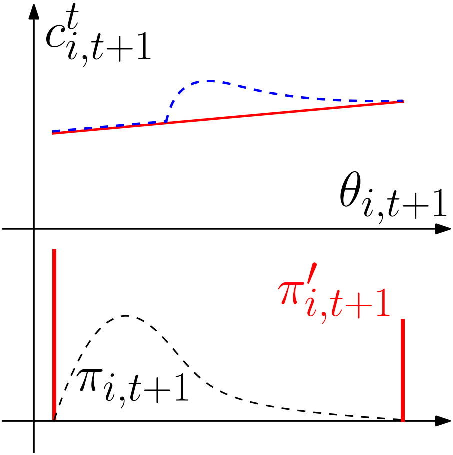
Affine optimal policies?

Lemma. At any t, any agent i weakly prefers affine $\overline{c}^t_{i,t+1}$ to nonlinear $c^t_{i,t+1}$ (affine in $\theta_{i,t+1}$)



Affine optimal policies?

Lemma. At any t, any agent i weakly prefers affine $\bar{c}^t_{i,t+1}$ to nonlinear $c^t_{i,t+1}$ (affine in $\theta_{i,t+1}$)



Affine optimal policies?

- **Necessary**: one-dimensional impact of shock $\theta_{i,t+1}$ on instantaneous utility
    - in example: inelastic labor, belief in independence

Affine optimal policies?

- **Necessary**: one-dimensional impact of shock $\theta_{i,t+1}$ on instantaneous utility
  - in example: inelastic labor, belief in independence

- Otherwise: affine **only in current utility**, $u\left(C_{t+1}^{t}\right)$

Affine optimal policies?

- **Necessary**: one-dimensional impact of shock $\theta_{i,t+1}$ on instantaneous utility
  - in example: inelastic labor, belief in independence
- Otherwise: affine **only in current utility**, $u\left(C_{t+1}^{t}\right)$
- Even when affine preferred, feasibility not guaranteed
  - modifying policy to above secant takes additional resources

Uncertainty in macro: example

Uncertainty in macro: example

- agent i at t has statistical model $\pi^*_{i,t+1}$ of θ_{t+1}
  (focus on θ_{t+1}, "too difficult" to eliminate any π_{t+2})

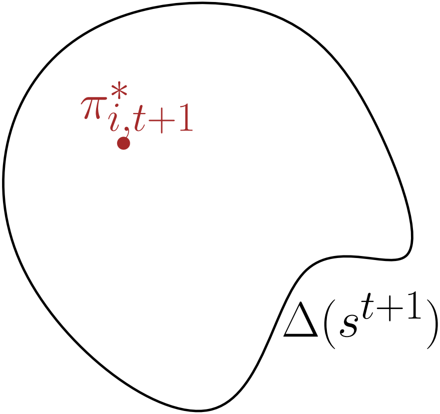
Uncertainty in macro: example

- agent i at t has statistical model $\pi^*_{i,t+1}$ of θ_{t+1}
  (focus on θ_{t+1}, "too difficult" to eliminate any π_{t+2})
- distrusts it, considers "nearby" models π_{t+1}:
  $$\Pi^{HS}_{i,t+1} \equiv \left\{ \pi_{t+1} \middle| \, d\left(\pi^*_{i,t+1}, \pi_{t+1}\right) \leq \epsilon \right\}$$

Uncertainty in macro: example

- agent i at t has statistical model $\pi^*_{i,t+1}$ of θ_{t+1}
  (focus on θ_{t+1}, "too difficult" to eliminate any π_{t+2})
- distrusts it, considers "nearby" models π_{t+1}:

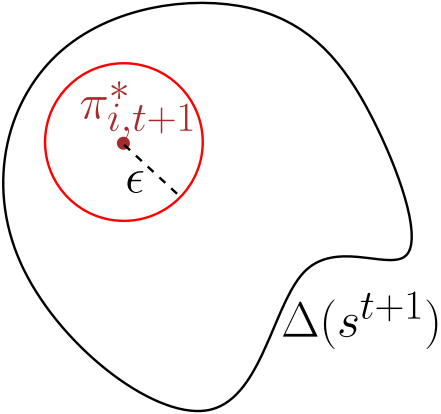
$$\Pi^{HS}_{i,t+1} \equiv \left\{ \pi_{t+1} \middle| \, d\left(\pi^*_{i,t+1}, \pi_{t+1}\right) \leq \epsilon \right\}$$



Uncertainty in macro: example

- agent i at t has statistical model $\pi^*_{i,t+1}$ of θ_{t+1}
  (focus on θ_{t+1}, "too difficult" to eliminate any π_{t+2})
- distrusts it, considers "nearby" models π_{t+1}:

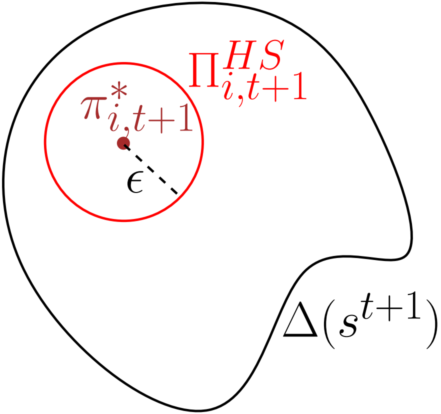
$$\Pi^{HS}_{i,t+1} \equiv \left\{ \pi_{t+1} \middle| \, d\left(\pi^*_{i,t+1}, \pi_{t+1}\right) \leq \epsilon \right\}$$



Uncertainty in macro: example

- agent i at t has statistical model $\pi^*_{i,t+1}$ of θ_{t+1}
  (focus on θ_{t+1}, "too difficult" to eliminate any π_{t+2})
- distrusts it, considers "nearby" models π_{t+1}:

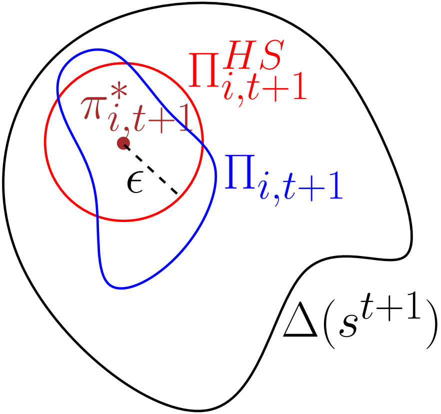
$$\Pi^{HS}_{i,t+1} \equiv \left\{ \pi_{t+1} \middle| d\left(\pi^*_{i,t+1}, \pi_{t+1}\right) \leq \epsilon \right\}$$



Uncertainty in macro: example

- agent i at t has statistical model $\pi^*_{i,t+1}$ of θ_{t+1}
  (focus on θ_{t+1}, "too difficult" to eliminate any π_{t+2})
- distrusts it, considers "nearby" models π_{t+1}:

$$\Pi^{HS}_{i,t+1} \equiv \left\{ \pi_{t+1} \middle| \, d\left(\pi^*_{i,t+1}, \pi_{t+1}\right) \leq \epsilon \right\}$$



Uncertainty in macro: example

- agent i at t has statistical model $\pi^*_{i,t+1}$ of θ_{t+1}
  (focus on θ_{t+1}, "too difficult" to eliminate any π_{t+2})
- distrusts it, considers "nearby" models π_{t+1}:

$$\Pi^{HS}_{i,t+1} \equiv \left\{ \pi_{t+1} \middle| \, d\left(\pi^*_{i,t+1}, \pi_{t+1}\right) \leq \epsilon \right\}$$