



Design of Debt Covenants and Loan Market Conditions¹

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Abstract

When a debt covenant is violated the lender has the right to demand immediate repayment of the loan. Using this right, the lender can extract certain concessions from the borrower (manager), which may be inefficient. I propose a theory that explains why, despite this inefficiency, tight and often violated debt covenants may be optimal. In a repeated moral hazard problem combined with an incomplete contract set-up, the debt overhang prevents the manager from exercising optimal effort. I deviate from the standard incomplete contract set-up by allowing outside market participants to observe the uncontractable outcome. I model the manager's outside option as the opportunity to refinance his debt on a competitive loan market. In this situation, the market independently evaluates the manager's performance based on observable parameters. The value of the outside option has an important impact on the covenant design. A strict covenant will severely punish the manager if his outside option is low. If the covenant is violated the lender will have control over the manager's assets and the manager will face a renegotiation game in which the lender has all the bargaining power. In this case a high outside option allows the manager to retain some rents. The manager will exercise effort to increase his chances to have a high outside option.

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1. Introduction

In this paper, we discuss the role of debt covenants in shaping future renegotiation. Debt covenants are a common feature of private credit agreements and are generally understood to protect lenders from opportunistic behavior by borrowers. Covenant violation gives the lender the right to demand the immediate repayment of the loan. Using this right the lender may extract concessions from the borrower. Recent empirical papers show that debt covenants are violated and lenders receive the right to withdraw the loan relatively often. In many cases, however, the lenders do not use the bargaining power that a covenant violation gives them and just forgive the violators. Ditchev and Skinner (2002) argue that "Lenders use covenants as "trip wires" which provide them with an option to step in and take action when circumstances warrant". I propose a new model of debt covenants that shows that the "trip wire" story, which is also suggested in other empirical works,² may be a part of a bigger picture where debt covenants are designed to affect the possibility of the renegotiation in the future.

I build the model on the presumption that contracts are incomplete. Incompleteness introduces distortions in actions taken by the borrower. Roberts and Sufi (2009b) show that long-term debt contracts are constantly renegotiated. Renegotiations arise when some information (like credit quality, investment opportunities etc) becomes available. The possibility of the renegotiation is beneficial as it re-introduces some completeness, which makes contracts more efficient. Foreseeing the possibility of the renegotiation the borrower will be taking actions that affect some uncontractable variables which may play an important role in determining the outcome of the renegotiation.

The downside, however, is that the borrower will not want to renegotiate if the predicted outcome of the renegotiation is unfavorable to him. The borrower may pre-commit to renegotiate by giving the bank one-sided power to renegotiate in those states of the world where the borrower would not do it ex-post. The mechanism of this pre-commitment is the covenant which the borrower is very likely to violate. Ex-ante this will be efficient as the degree of market completeness is higher. Because of the contract incompleteness the covenant cannot perfectly determine the state of the world and the violation is possible in good states of the world as well. However, that does not matter as the borrower himself will initiate the renegotiation in those states.

² See, for example, Chava, Roberts (2008) and Roberts, Sufi (2009a)

One of the contributions of the paper is that it shows that the choice of the ex-ante probability of the covenant violation is non-trivial. Ex-post the lender may use the threat of the loan withdrawal to destroy the value of the firm by demanding too much of costly concessions from the borrower. The optimal probability balances the positive effect described above and the negative effect of the inefficient lender's control. I will refer to this probability as "covenant tightness" (or "strictness"). To my knowledge, this is the first paper that provides a framework which allows us to analyze the determinants of the optimal strictness of debt covenants.

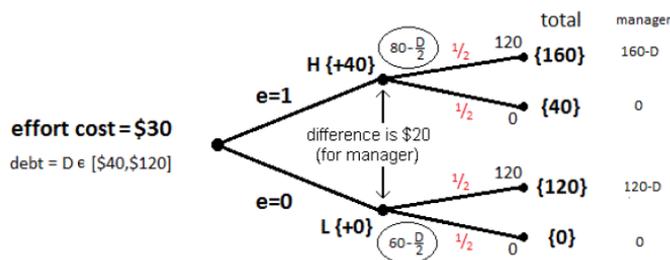
The details of the model are as follows. The manager takes a loan from a bank to start his project. The project consists of two stages: the preliminary stage and the final stage. The manager exercises effort both at the preliminary stage and at the final stage. Each stage generates a return. The efforts and the returns are independent across time: the effort at the preliminary stage affects only the return at the preliminary stage, but not the return at the final stage. The effort at the final stage affects only the return at the final stage of the project. All the returns are realized at the end and the total return is the sum of the two. Crucial assumption is that the soft information of the return at the preliminary stage is available earlier.

The long-term debt contract creates inefficiencies because of the debt overhang and the limited liability: the manager does not exercise optimal effort as part of the proceeds goes to the bank. The repeated structure makes the problem much worse because from manager's point of view the effort exercised at the preliminary stage can be wasted. If something does not work out at the final stage and the total return is less than the debt, the firm is in bankruptcy and the manager retains nothing. In this unfortunate situation, everything that manager's effort produced at the preliminary stage goes to the bank as a form of collateral. Foreseeing that, the manager exercises less than optimal effort at the preliminary stage.

As a simple example consider the following toy model. The preliminary stage result may be either state H or state L , where H indicates that extra \$40 will be available later. To get state H the manager needs to exercise effort at private cost \$30. If the effort is not exercised state L occurs. The final stage of the project gives return \$120 with probability $\frac{1}{2}$. This makes the total return being the lottery: \$160 with probability $\frac{1}{2}$ and \$40 with probability $\frac{1}{2}$ if effort is exercised and \$120 with probability $\frac{1}{2}$ and \$0 with probability $\frac{1}{2}$ if effort is not exercised. If the

manager has debt with a face value $D \in [40,120]$ he is bankrupt whenever the final project results in a failure no matter whether he exercised the effort or not. The difference for him between his expected earnings at state H and at state L is just \$20,³ and his choice will be not to exercise effort, which is inefficient. The game is shown on Figure 1

FIGURE 1: Toy Model



By itself, the possibility to renegotiate the agreement does not make a difference because the bank will hold up the renegotiation after the manager has already exercised the effort. However, if the manager has an outside option that depends on the state H or L , there can be a more efficient contract. Assume the following. 1) To start the project the manager needs to borrow initial investment $I = \$50$. 2) The contract may specify two options for the manager: early repayment amount, E and final payment $D > E$. The manager may choose to pay E after the preliminary stage is over or to pay D after the whole project is over. 3) There are competitive banks that are willing to refinance the manager (to pay E to the initial lender) on fair terms after the preliminary stage is over. They see the state of the world: H or L . 4) Everyone is risk-neutral and risk-free interest rate is zero.

Then, 1) the contract $E = 50, D = 100$ will be accepted by a bank. 2) If state is H the contract will be renegotiated and final debt will be $D^R = 60$. 3) If state is L the contract will not be renegotiated. 4) The manager exercises effort which is efficient.

If the state of the world is H an outside bank knows that the manager will earn at least \$40, and is ready to refinance under the condition that the manager must pay \$60 after all returns are realized. If the final stage of the projects turns out to be a success the manager pays \$60 and keeps \$100. If the final stage results in zero return the manager pays just \$40. The new lender breaks even in expectation, the manager expects to get \$50 if he exercises effort and the state is

³ His expected earnings are $80 - \frac{1}{2}D$ in state H , and $60 - \frac{1}{2}D$ in state L

H. If the state is *L* an outside bank may only suggest the refinancing on the same conditions as the manager was financed initially. The manager expects to get \$10 if no effort is chosen and state *L* is realized. The difference, \$40, compensates the manager's choice of effort which results in the efficient action taken by the manager.

This example shows how the possibility of the renegotiation may re-introduce contingencies back into the model with incomplete contracts. This fact by itself is not surprising. By assuming that an important uncontractable variable may be observable by all market participants I suggest a setting which is between the classical incomplete contract framework and the complete contract set-up. The non-triviality of this approach comes from the asymmetric rights to renegotiate. In the current set-up, the bank is committed to the initial terms of credit, while the manager can quit at any time by refinancing on better terms with another bank. If outside option is worse, however, the manager will prefer to stay with his initial agreement and because of the debt overhang the ex-ante effort will be inefficient. The renegotiation favors the manager in good states of the world and favors this bank in bad states. The role of debt covenant is to increase the difference for the manager between good and bad states and, therefore, provide ex-ante incentives.

Assume that in the settings described above the refinancing requires a costly state verification by an outside bank. The cost \$15 lowers the manager's outside option as he should compensate for this spending. Assume that the contract still requires early payment $E=50$ or final payment $D=100$.⁴ In this case if state is *H* the manager's outside option is \$35 ($\$50-\15), and the option to keep the agreement gives him \$30 ($=\frac{1}{2} \cdot (160 - 100) + \frac{1}{2} \cdot \max\{0,40 - 100\}$). In state *L* the manager does not have an outside option ($\max\{\$10 - \$15,0\} = 0$), and his "inside option" is \$10 ($=\frac{1}{2} \cdot (120 - 100) + \frac{1}{2} \cdot 0$). If the state is *H*, the manager picks his option to renegotiate because his outside option is better than his inside option, and he expects to get \$35. In state *L* the manager does not want to renegotiate because he has no outside option and he expects to get \$10 by keeping his initial agreement. The difference \$25 does not cover his effort costs \$30, so the inefficient action is chosen.

⁴ In the new settings, this contract is not an equilibrium contract any more. As early repayment is not realized in the equilibrium, the parties may agree on $E=35$ which bring us back to the original outcome. The full analysis that takes this into account is conducted in the main section. The analysis here is just an illustrative example.

If the manager can commit to renegotiate by accepting a very strict debt covenant he will be afraid of the state L because in this state his outside option is very low. A covenant violation gives the bank the right to demand the immediate repayment of the loan, or, in the current setting, the manager loses his option to pay $D=100$ late and has to pay $E=50$ early. Since he does not have an outside option the bank has all the power at the renegotiation and can increase the final payment to the maximum reasonable value: $D^R = 120$. If the covenant is always violated the expected outcome for the manager in state L is $\$0$. The difference between states H and L is $\$35 > \30 , and the optimal effort will be exercised in the equilibrium.

From this simple example, we learn that debt covenant violations not only allow the bank to change the initial terms of agreements, but also may provide ex-ante incentives. Ex-post, however, tight debt covenants may be detrimental. If the final stage of the project also requires effort, this effort will be the lower the higher is the manager's debt before the beginning of the final stage. If a covenant is violated and manager's outside option is low the lender may use his bargaining power to increase the face value of the debt which increases bank's revenue but decreases manager's effort and efficiency. The incentive effect that debt covenants produce should be weighed against the detrimental effect of tightening them.

The model allows one to explain recently found empirical facts about debt covenants. Several empirical papers show that covenants are remarkably tight. According to the analysis by Chava, Roberts (2008), the covenant on the Current Ratio is set just below the value of the Current Ratio variable at the time of signing the agreement: on average 1.09 standard deviations from it. The Net Worth covenant threshold is even tighter: 0.68 standard deviations away. This leads to many violations. Roberts and Sufi (2009a) document that more than 30% of publicly traded firms violate a debt covenant at some point in their life. Chava, Roberts (2008) and Ditchey, Skinner (2002) show that 15-20% (depending on the type of covenant) of outstanding loans are in violation during a typical quarter.

Roberts and Sufi (2009a) also show that lenders do not always use the bargaining power that the covenant violation gives them. In more than half of the cases covenant violations are routinely waived without any consequences to the firm. Chava, Roberts (2008) also find that for large groups of violators the ex-post consequences are insignificant. My model suggests that lenders might not have the bargaining power in those situations because the outside option of the manager might be high. This is partially confirmed by the empirical results in the Roberts,

Sufi (2009a) paper. They show that variables: Leverage ratio, Market-to-book ratio and whether the firm has the S&P credit rating significantly affect the consequences that the violator experiences.

Given the number of covenant violations and the ex-post reaction it is reasonable to suggest that covenants do not always capture the most important variables in the set-up. The model suggests that debt covenant violations by themselves do not have to be a bad signal. It is more efficient if parties can set debt covenants on important variables and "trap" only firms that are in bad state, however due to contractual incompleteness it is not always possible. The model suggests that even if the noise between the variable we can set a debt covenant on (a contractible variable) and the variable we really care about is significant, it is still possible there will be a debt covenant. In the simple example above the variable that lenders really care is the state of the world $\{H, L\}$, while the contractible variable is absent. This is equivalent to the noise between the contractible and the uncontractable variable being infinity.⁵

Except for Murfin (2012)⁶ current literature mostly focuses on borrower's characteristics to determine the degree of covenant tightness. While they are important the loan market conditions should also play a role. My model allows one to derive comparative static properties with respect to the loan market conditions. If the markets are good the outside option may be high if the realization of the preliminary stage is very good and "OK" for poor realization of the preliminary stage. Covenants might not be needed in this situation as the manager will very likely choose the option to renegotiate. Once outside option worsens the benefits of the renegotiation decrease uniformly (in both good and bad states) and a possibility of punishment through tight covenants becomes available and useful. Therefore, when loan market conditions are expected to be bad, parties use stricter debt covenants ex-ante. There are anecdotal evidences that covenants are tighter during recessions and are loose during booms. Justin Murfin (2009) says "During the easy credit period from 2002-2006, for example, covenants were abandoned en masse. Since then, contracts have swung the other way, with financial trip

⁵ This is not the first paper that suggested that covenants may be random or close to being random. Sridhar and Magee (1997) model a debt covenant as a restriction on a financial variable which is random. They focus on the connection between the informativeness of this contractible variable and the tightness of the covenants. While this is a very interesting question and my model allows for an extension that could address it, this is not the focus of this work. A similar idea of uncertainty introduced by debt covenants was recently explored in Demerjian (2015)

⁶ He empirically shows that covenant tightness also depends on shocks that particularly affect a given lender. If one of his loans defaulted the next borrower will receive a tighter covenant.

wires set such that lenders receive contingent control rights for even modest borrower deterioration." This is roughly consistent with the predictions of my model.

Theory and evidence suggest that riskier firms on average receive contracts with stricter covenants. Dichev, Skinner (2002) found that covenants are stricter for highly leveraged firms. This conclusion may also be derived from Chava, Roberts (2008) and Roberts, Sufi (2009a) who analyze ex-post reactions to covenant violations and find that riskier firms suffer more. Theoretical paper by Garleanu and Zwiebel (2008) analyze the debt covenant design when there is a possibility of future transfer from the bank to the manager, and the manager is better informed about it. The covenant is tighter if the difference between information sets is bigger. In my model, there is an inefficient project which is not chosen in the equilibrium but may provide a competition with the main project. The contract must create incentives for the manager to choose the efficient project. The better this project is seen by the manager, the tighter debt covenants must be used. The predictions of my model are consistent with the findings that riskier firms receive tighter debt covenants.

My model also allows one to analyze a situation where there is a large negative shock to predicted loan market conditions. Assume that the manager can find financing now, but there is no possibility of refinancing in the future even if the performance is very good. Assume that this is known ex-ante. Leaving aside the issue of bank's bargaining power at this stage we can analyze the tightness of debt covenants. When the outside option does not exist the usefulness of covenants to motivate the manager is much lower. The parties will not be using tight debt covenants if future refinancing possibilities are terrible. This prediction is unique to my model and opens an interesting direction for future empirical work.

The traditional view of debt covenants is that they are designed to protect lenders from opportunistic behavior by borrowers. Tirole (2006) summarizes the two roles that debt covenants may play. The first role is to prevent managers and shareholders from taking value reducing activities that could privately benefit them. Smith and Warner (1979) discuss limiting dividends payouts. Berlin, Mester (1992) suggest that covenants limit investment in risky activities. Berlin, Mester (1992) also discuss the possibility of renegotiating the restrictive covenant in the good state of the world. The second role of covenants is to define circumstances under which the lenders take control over the firm. Aghion, Bolton (1992) show that it is optimal to allocate residual control rights to debt holders in bad states of the world.

This paper is different in the way that it treats debt contract and covenants as a particular attribute of them as a precursor for the renegotiation in the future. In that way, it is similar to Hebrman and Kahn (1988) who show that debt contracts may be designed so that the parties end up in a Pareto-inefficient outcome which they renegotiate.

Generally, the renegotiation is seen in the literature as something parties would like to avoid ex-ante to make a punishment credible. Stiglitz and Weiss (1983) propose a setting under which an efficient outcome may be reached if the lender commits not to sign an agreement in the future with a borrower who defaulted. Once we are in the future, however, it is efficient to "forget" about this commitment and sign the new agreement. Ex-ante the possibility to renegotiate negatively affects incentives.

This paper presents a different view on the renegotiation. By allowing other market participants to see the uncontractable variable I suggest that the possibility of the renegotiation may increase efficiency ex-ante. The paper presents the renegotiation as a move along the Pareto frontier. The major issue here is who can renegotiate and under what conditions. I show that debt covenants might play an important role in determining those conditions.

The paper is also related to debt structure and debt maturity. Jensen and Meckling (1977) introduced debt overhand problem in a dynamic model of financing. The source of problem, the separation of ownership (bank) and control (manager) is also a feature of the model presented here.

The remainder of the paper is organized as follows. Section 2 describes the model set-up. In section 3 I present the solution of a simplified model, where I treat the parameter that specifies early repayment of the loan as exogenous. In section 4 I provide the solution to the full model and illustrate the results with a numerical example. I discuss comparative statics in section 5. Section 6 concludes. All the proofs are in the technical appendix that is available on request.

2. Model set-up

The toy model presented in the introduction captures many important features of the main model. It does not, however, capture the debt overhang effect at the final stage of the project. For that we need to introduce a more complex structure which includes manager's exercising effort at the final stage as well. I also need to introduce a noise in the first period outcome. In the main model the effort in the first period will not guarantee high outcome, but will increase the probability of it. Also, the outcome of the project at the preliminary stage is continuous. Loan market conditions are modelled as the opportunity of use of funds for lenders who compete for refinancing the firm.

2.1 Description

There are two periods in the model. A financially constrained entrepreneur (manager) needs to finance the start-up cost I of his project. There is a competitive lending market and a bank finances the manager on the condition of breaking even in the expectation. Both the manager and the bank are risk neutral.

There is a predicted change in the loan market conditions. Before period 2 starts the manager will be able to refinance the project with another lender on a competitive market. The opportunity costs of funds are predicted to be r^O ($r^{outside}$). This, however, does not affect the opportunity cost of funds for the current (initial) lender, which I normalize to zero.⁷

After the contract is signed and the initial investment I is made, the manager has a choice between two projects.

The first (efficient) project gives returns that are realized in the period 2. The returns y_1 and y_2 are realized at the same time, however information on y_1 is available earlier, in period 1. All market participants have access to this information. The project requires efforts both in period 1 and in period 2; the effort in period 1 affects y_1 , but not y_2 , and the effort in period 2 affects

⁷ We may think that each individual bank has its own opportunity costs, and they do not necessarily have to be correlated with each other. We can normalize to 0 opportunity costs of the initial lender and interpret r^O as the best interest rate the manager can get outside of his credit agreement at the time he'll want to refinance.

only y_2 . Important assumption is that the observed information about the variable y_1 cannot be included into the contract.⁸

We can think of any two-staged project where some results of the first stage are observable earlier, but final returns are realized only after the second stage. The contract cannot be made directly on the earlier observed results. For example, take an advertising campaign that precedes sales organization. Market participants can see the campaign and can form conclusions about expected return, however it's hard to imagine a contract that explicitly specifies expected effect of the campaign on the sales.

The second project represents manager's ability to get involved in a different activity. This is an inefficient activity that provides a competition to the main project. It does not require effort. It gives fixed return y_0 in period 1. One of the goals of the initial contract is to create incentives for the manager to choose the efficient project.

I assume that at the time the information on y_1 is observable another variable is realized, M_1 , which is a noisy signal of y_1 . The difference between y_1 and M_1 is that M_1 can be included in the contract, while y_1 , although observable, not contractible. The contract may include a covenant that is conditioned on M_1 . In my model, M_1 can take two values: $\{M_g, M_b\}$, indicating for the good and the bad state. The covenant is modeled as the probability that the bank will have the control over the manager's assets and will be able to demand early repayment of the loan.

The level of the correlation between variables y_1 and M_1 is described by the variable ρ .

$$Prob\{M_1 = M_g\} = \frac{1}{2} \left(1 + \rho \cdot \frac{y_1}{y_H} \right),$$

where y^H is the maximum value of y_1 (the minimum value of y_1 is zero).

If $\rho > 0$ the variable M_1 is informative. Otherwise (if $\rho = 0$) it is a coin toss that is completely irrelevant to the model. Probabilities that the bank will have control in the good state or in the bad state are parts of the contract. Notation is P_{good} and P_{bad} respectfully. Probability that the bank will have control conditional on y_1 is

⁸ The possibility of a variable to be observable but not contractible is discussed in the huge literature on the incomplete contracting that started with Grossman and Hart (1986)

$$P_{contr}(y_1) = \frac{P_{good} + P_{bad}}{2} - \rho \cdot \frac{y^1}{y^H} \cdot \frac{P_{bad} - P_{good}}{2}$$

where $\frac{P_{good} + P_{bad}}{2}$ can be interpreted as the average tightness of the covenant, and $\frac{P_{bad} - P_{good}}{2}$ dispersion. The term $\rho \cdot \frac{y^1}{y^H}$ captures the parties' ability to contract on what is happening after the contract is signed, namely, y_1

Although the possibility to contract on variables connected to y_1 is important, the concentration of the paper is to analyze the covenant tightness, which is the first term of the formula above. I show that even in the sharpest degree of the contractual incompleteness ($\rho = 0$) there will be a non-trivial probability that the bank will have control. In this paper, I will analyze only the special case: $\rho = 0$. The covenant in the contract will be defined as P_{contr} .

2.2 Timeline

The timeline of the events is as follows:

- Period 0
 - The parties are informed about future market conditions r^0
 - The contract is signed, the face value of the debt, D , is specified, early payment E is specified
- Period 1
 - The manager makes a choice between the efficient project and the inefficient one. If the inefficient project is chosen, the return y_0 is realized, the payment E is made and the game is over. Otherwise:
 - The manager exercises the effort that affects the distribution of y_1
 - The information about y_1 is observed by everyone
 - M_1 is realized (although not important if $\rho = 0$)
 - The manager can initiate the renegotiation process. (He will do it if he has a good outside option.)
 - The covenant is checked. If it is violated the bank may initiate the renegotiation process. (The bank will use it if manager's outside option is bad.)
 - If the renegotiation was initiated by either party, the new agreement is signed, the parties specify the new face value of the debt, D . Otherwise they follow the old agreement.

- Period 2
 - The manager exercises effort that affects distribution of y_2
 - y_1 and y_2 are realized
 - Final payment is made

2.3 Technology assumptions

Assumption T1. y_1 is continuous with a distribution function that depends on manager's effort in the first period, which is costly

$$y_1 \in [0, y^H]$$

$$y_1 \sim G(\cdot, e_1)$$

High effort is better than low: $G(\cdot, e_1^1)$ First Order Stochastically Dominates $G(\cdot, e_1^2)$ if $e_1^1 > e_1^2$; $e_1 \geq 0$, density of y_1 is $g(\cdot, e_1)$.

Assumption T2. Effort cost in the first period, $C_1(e_1)$, is an increasing and convex function with $C_1(0) = 0$; $C_1'(0) = 0$, and $C_1'(1) = \infty$.

Assumption T3. y_2 may take two values: y^S (success) or 0 (failure). Probability of success depends on manager's effort in the second period, which is costly. For simplicity let the probability of success to be *equal* to effort

$$y_2 \in \{0, y^S\}$$

$$\Pr\{y_2 = y^S\} = p_2 = e_2$$

p_2 is bounded below and above by p_l and p_h : $p_2 \in [p_l, p_h]$.

Assumption T4. Effort cost in the second period, $C_2(p_2)$, is increasing and convex; $C_2(p_l) = 0$; $C_2'(p_l) = 0$.

Assumption T5. Higher effort is socially better: $p_2 y^S - C_2(p_2)$ is increasing in p_2 .

Assumption T6. Technical assumption (normalization): $C_2'(p_h) = y^S$

Assumption T7. Bank's "moral hazard": $D \cdot C_2'^{-1}(y^S - D)$ is increasing in D

Assumption T8. Risk-averse attitude

$$\frac{\partial^3 C_2(p_2)}{\partial p_2^3} < \left(\frac{\partial^2 C_2(p_2)}{\partial p_2^2} \right)^2$$

Assumptions T1 and T2 imply that the effort in the first period is good, and there will be some socially optimal effort $e_1^* \in (0,1)$ that maximizes expected return minus cost. Assumptions T3 and T4 guarantee that as manager's debt increases (as his share of the output y^S goes down), the effort will be lower. Assumption T5 states that effort in the second period is good, implying, together with assumptions T3 and T4, that best possible situation is when the manager gets all of y^S in case of success. Assumption T6 says that the effort will be lower whenever the manager has an outstanding debt after the adjustment for the guaranteed future income y_1 (The second period effort will be the optimal if y_1 fully covers the debt that the manager has to pay to the bank. It will be less than the optimal otherwise)

Assumption T7 guarantees that the incentives of the manager and the bank will be opposite. From previous assumptions, we understand that while the bank increases the face value of the debt the manager exercises less effort. To introduce divergence of incentives we must assume that the increased debt will compensate the bank for the lower probability of success. At the time of the renegotiation the bank, acting opportunistically, will be trying to destroy the value of the firm by increasing the interest rate.

Assumption T8 will guarantee that ex-ante the parties will be afraid of the risk that is carried by the second period project. It is important to understand that the risk-averse attitude will be about the realized manager's effort in the second period which is connected to the final debt. The parties will not want a lottery between a very high debt (which will effectively result in the inefficient liquidation)⁹ and a low debt. Instead they'll prefer an average debt with.

⁹ An extremely high debt will result in a scenario that can be interpreted as a liquidation because the resulting manager's effort will be at a lowest bound. There will be a positive probability of success, $p_2=p>0$. Bank's expected earnings of the "liquidated" firm are py^S .

2.4 Renegotiation assumptions

Assumption R1. The manager may initiate the renegotiation after the information about y_1 is available.

Assumption R2. The bank may initiate the renegotiation when the manager violates the covenant.

Assumption R3. The bank has all the bargaining power at the renegotiation

Since initially the manager offers the terms of the agreement to the bank, it is plausible to allow him as much flexibility as possible. At the same time the hold-up issues need to be accounted for. After signing the agreement with a particular bank the manager has much less bargaining power than he had before, so, the assumption that the bank has full bargaining power at the renegotiation is plausible.

As soon as the information about y_1 is available the manager has an outside option. This is a common knowledge and the manager can always credibly threaten to take it. In some states of the world the outside option is bad for the manager. Then he will never take it ex-post, but he may always pre-commit to take it through the covenant. He will do this when this pre-commitment increases ex-ante efficiency.

2.5 Contract space

The contract specifies the covenant which is modeled as the probability that the bank has the control, P_{contr} , the final payment of the debt, D , and the possibility of the early repayment, E .

The contract is signed with the face value of the debt D , but it allows for the early repayment at manager's choice.

The covenant determines the level of control that the lender will have. As discussed in the introduction, covenant violations happen often, in many cases lead to a waiver. To explain

stylized facts, we need to allow for a covenant violation in the model. The simplest way to do it is to assume that the covenant is violated with certain probability. In other words, the parties may specify the probability that the bank will have control, P_{contr}

Assumption A1. There exists an upper-bound value of the probability of bank's control

$$P_{contr} \leq \overline{P_{contr}} < 1$$

This is a technical assumption to ensure the variable D is important. As will be shown later, the manager picks the outside option whenever it is better for him, and the bank -- whenever it has the rights to do so (covenant violated, the event that has probability P_{contr}), and whenever the outside option is worse for the manager. So, D (initial face value of the debt) matters only if outside option is worse for the manager and there is no covenant violation. With this assumption, I want to ensure that the dependence on D will not disappear.

2.6 Outside option

After the information about y_1 is available the manager may pick the option to repay the debt early. As he has no cash at this point, he needs to refinance the debt with another bank. This bank pays E to the initial lender and offers a new agreement to the manager. This agreement specifies interest rate \tilde{r} . The manager chooses the effort in the second period considering his earnings after repaying the debt to the new lender.

$$\max_{\tilde{p}_2} \{ \tilde{p}_2 \cdot \max(y^S + y_1 - (1 + \tilde{r})E, 0) - C_2(\tilde{p}_2) \}$$

The lenders are competitive and the opportunity cost of funds $(1 + r^0)$. If $y_1 \geq (1 + r^0)E$, this debt is risk-free with interest rate $\tilde{r} = r^0$. Otherwise the break-even condition for the lenders is

$$y_1 + \tilde{p}_2 \cdot ((1 + r) \cdot E - y_1) = (1 + r^0) \cdot E$$

If refinancing is possible, the manager's outside option is

$$V_2^0(y_1, E, r^0) = p_2^0 \cdot (y^S + y^1 - (1 + \tilde{r}^*) \cdot E) - C_2(p_2^0)$$

where p_2^0 and \tilde{r}^* solve the two equations above

If the refinancing is impossible (the debt is too costly), the outside option is zero. Assumption $r^0 > 0$ guarantees that the outside option will never be picked in the equilibrium: the initial lender will always be better off by offering the same terms of the agreement as the outside lender than by accepting the early payment E .

2.7 Additional assumptions

Two more assumptions are required to avoid trivial and non-interesting solutions

Assumption A2. No contract with face value of the debt $D \leq y^H$ is feasible. If $B(D, E, P_{contr})$ is the value of the contract for the bank, the assumption is equivalent to

$$\max_E \max_{D \leq y^H} B(D, E, P_{contr}) < I$$

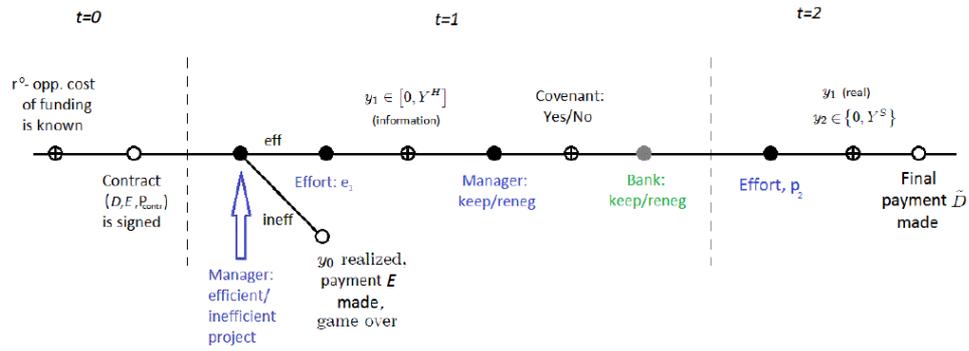
Assumption A3. No contract such that the outside investors can be paid from the realization of y_1 is feasible: $y^H < E(1 + r^0)$

The assumptions ensure that the manager enters the second period with certain debt that cannot be covered from the guaranteed first period earnings. He must work through the success of the second project in order to retain some positive rent.

2.8 Summary (timeline)

Figure 2 summarizes the full timing of the event

FIGURE 2: Timeline



First parties learn market conditions shock r^0 . Then they sign the agreement that specifies the payments and the debt covenant. After the agreement is signed the manager chooses the project. He may choose the efficient project or the inefficient one. If the efficient project is chosen the game goes on until the final payment D^f is made. Otherwise, y_0 is realized and payment E is made. D^f is either the initial debt, D , or the renegotiated debt ($D^f = D$ if there was no renegotiation). The black dots on the figure indicate points where the manager makes a decision. The grey dot -- bank's decision. The crosses indicate realizations of the exogenous variables. White dots -- the original contract and the final payments.

3. Simplified model

The purpose of this section is to show the main trade-off behind the debt covenant design: the trade-off between the incentives that the covenant provides and the bank's moral hazard. To explore it I fix the ex-ante outside option. There are three parameters that the outside option depends on: opportunity cost of funding for "outside" banks (r^0), contractual early repayment amount (E), and the realization of y_1 . For this section I assume that the early repayment E is pre-specified. r^0 is also exogenous, so the outside option depends only on the realization of y_1 . I also assume that y_0 is small and that the inefficient project will never be chosen.

Let $V(D, P_{contr}, E, r^0)$ be manager's expected value of the contract (D, E, P_{contr}) ; $B(D, P_{contr}, E, r^0)$ -- bank's expected value. The managerial problem is

$$\begin{aligned} \max_{D, P_{contr}} & V(D, P_{contr}, E, r^0) \\ \text{s. t.} & B(D, P_{contr}, E, r^0) \geq I \end{aligned}$$

The model is solved by backward induction.

3.1 Road map

- Detailed discussion of the second part of the project
 - Lemma 1:
- Backward induction logic
- Formulation of preliminary results through a set of lemmas
 - Lemma 2:
 - Lemma 3: advantages of the outside option
 - Lemma 4: use of debt covenants as motivation
- Formulation and discussion of the main result

3.2 Second part of the project, $t=2$

The starting point is to solve the second stage of the model. At $t = 2$ the manager has certain debt, D^f which is either D or the renegotiated debt. Define $\tilde{D} = D^f - y_1$. Then $y^S - \tilde{D}$ will be manager's share of the second project if it is successful. Manager's surplus is

$$V_2(\tilde{D}) = \max_{p_2} (p_2 \cdot (y^S - \tilde{D}) - C_2(p_2))$$

Bank's surplus: $B_2(\tilde{D}) = p_2(\tilde{D}) \cdot \tilde{D}$ where $p_2(\tilde{D})$ is the solution of the equation above. Notice that it does not include y_1 as it is already guaranteed. The bank gets y_1 if the project on the second stage fails and $D^f = \tilde{D} + y_1$ if it succeeds. A few important observations about the second stage of the project are summarized in Lemma 1.

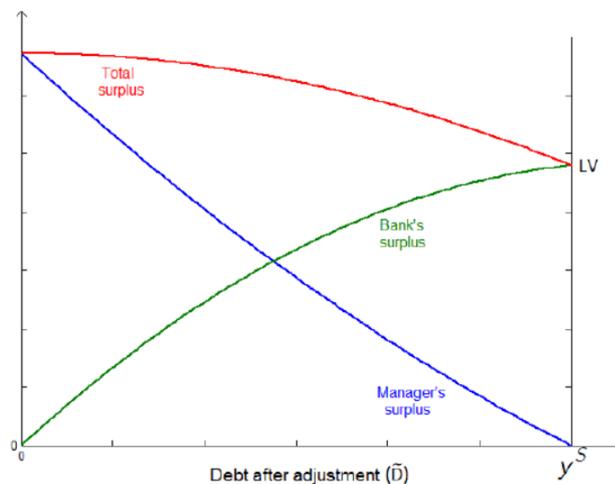
Lemma 1.

- 1) There is a debt overhang. The higher the debt is the lower the manager's effort is.
- 2) Bank's control worsens debt overhang. Ex-post the bank prefers higher interest rate.
- 3) The parties are risk-averse regarding the second stage project. Any lottery that gives the bank certain expected value would be more preferred to the manager than a mean-preserving spread of that lottery.

The theory in sections 4 and 5 is illustrated by a numerical example which I will refer to "the leading example". Functions and numerical values for the leading example in the paper are picked as follows: $g_1(y_1, e_1) = \frac{2}{y^H} (1 - e_1) - \frac{2y_1}{(y^H)^2} \cdot (1 - 2e_1)$, $y^H = 40$. $y^S = 120$; $p_l = 0.4$, $p_h = 0.45$; $C_1(e_1) = A_1 e_1^2$; $C_2(p_2) = A_2 \cdot (p_2 - p_l)^2$; $A_1 = 0.889$; $A_2 = 1200$; $I = 50$, $y_0 = 65$. Whenever applicable, specific parameters: $D = 106$; $E = 44$.

Figure 3 illustrates the lemma 1 statements.

FIGURE 3: Illustration for lemma 1



The sharp decreasing function illustrates manager's share of surplus of the second part of the project as a function of "adjusted debt", \tilde{D} . If the debt is very big ($\tilde{D} = y^S$) the manager's surplus is zero and effort is at the minimum level. The increasing function is bank's surplus (bank's expected revenue minus "first period earnings", y_1). The last function, the sum of the two, is the total surplus of the second part of the project.

The second part of the project captures the bank's possibility to damage firm's value or even liquidate the firm. The bank, acting opportunistically, will be increasing the face value of the debt (or interest rate) up to the point where the manager is indifferent between accepting new terms of the agreement or refinancing. The higher is the debt the lower is the manager's share of the outcome y^S in case of success, the lower is the effort. In the extreme case, the manager should pay back all the output in case of success, and the resulting effort will be zero. This may be interpreted as the liquidation.

Liquidation value is the minimum value of the firm. The firm is "liquidated" if the adjusted debt equals total output of the second part of the project, y^S . In this situation, the manager's surplus is zero. It is possible only if manager's outside option is zero. The bank, having all the bargaining power at the renegotiation, will suggest an agreement where the manager pays everything and the manager must accept it as he has no outside option. I assume that once the manager started this project he does not have the power to destroy it, so the minimum value of the second part of the project is the liquidation value, $LV = p_l \cdot y^S$.

If the debt is lower, the effort is higher, the total surplus is higher and the manager's surplus is higher. However, the bank will not voluntarily agree to lower the debt as the bank gets less thus. I assume that at the renegotiation the bank has all the bargaining power, so, whenever there is a renegotiation that is suggested (or, better to say, "enforced") by the bank the value of the firm gets destroyed. This is the inefficiency of giving the bank too much bargaining power through the covenants.

Assumption T8 guarantees that the total surplus function is concave. This implies that the parties are risk-averse regarding the adjusted debt at the beginning of the second period. Ex-ante they prefer a contract that results in less variability of \tilde{D} . Given the structure of the model this is natural, as high debt may result in liquidation and the parties ex-ante want to avoid it even if it happens with a low probability.

3.3 Backward induction

The next step is to define the manager's and bank's continuation payoffs for the two cases. First case is where the parties enter the second period keeping the initial agreement. The values depend only on the initially agreed debt, D , and the realization of y_1 . Let's call $V_2^K(y_1, D)$ – manager's value and $B_2^K(y_1, D)$ – bank's value. Second case is where the parties renegotiate the initially agreed payment D . The outcome of the renegotiation depends only on the manager's outside option. Ex-post, the bank is trying to keep the manager at the minimum utility level. For the manager, the renegotiation is resulted in getting his outside option. Let's call $V_2^R(y_1, E, r^0)$ manager's value at the renegotiation, and $B_2^R(y_1, E, r^0)$ – bank's value. $V_2^R(y_1, E, r^0) = V_2^O(y_1, E, r^0)$.

Once those continuation values are known we can deduce the bank's decision rule. The bank is opportunistic ex-post. It will insist on the renegotiation of the covenant if and only if it gets more from the renegotiation than otherwise, or, if $B_2^R(y_1, E, r^0) > B_2^K(y_1, D)$. After the realization of y_1 the manager himself may pick the option to renegotiate. He will do so if he has the outside option which is higher than the option of keeping the agreement: $V_2^R(y_1, E, r^0) > V_2^K(y_1, D)$. Lemma 2 shows that the bank's decision rule and the manager's decision rule are exactly the opposite. The bank wants to renegotiate whenever the manager wants to keep the agreement; the manager wants to renegotiate whenever the bank wants to keep the agreement. Lemma 3 shows that $V_2^R(y_1, E, r^0)$, as a function of y_1 , grows significantly faster than $V_2^K(y_1, D)$. The corollary from lemma 3 has crucial importance for the computation of the continuation values. It says that there exists a threshold value y such that for $y_1 > y_h$ and only for those y_1 the manager picks the option to renegotiate.

Given that we can compute the manager's and the bank's continuation values after the realization of y_1 : $V_1^{contr}(y_1, D, P_{contr}, E, r^0)$ and $B_1^{contr}(y_1, D, P_{contr}, E, r^0)$. Then we go one more step back to integrate the manager's continuation value over y_1 knowing effort e_1 . This way we get $\tilde{V}_1(e_1, D, P_{contr}, E, r^0)$. Making another step back we compute optimal effort (given effort costs) and, finally, manager's value of the contract: $V(D, P_{contr}, E, r^0)$. Knowing effort that the manager chooses we can compute the bank's value of the contract: $B(D, P_{contr}, E, r^0)$ by integrating $B_1(y_1, D, P_{contr}, E, r^0)$ over y_1 with the density function $g_1(y_1, e_1)$.

The last stage is to choose (D^*, P_{contr}^*) such that the two conditions are satisfied:

- 1) the banks at least break even: $B(D^*, P_{contr}^*, E, r^0) \geq I$, and
- 2) the manager gets maximum possible value: there is no other $(\tilde{D}, \widetilde{P_{contr}})$ that satisfies 1) and gives the manager better value $V(\tilde{D}, \widetilde{P_{contr}}, E, r^0) > V(D^*, P_{contr}^*, E, r^0)$.

3.4 Preliminary results

We will go over a set of lemmas which will help us to formulate the main result in the next section,

Lemma 2. The manager will want to keep the agreement whenever the bank will want to renegotiate. The manager will want to renegotiate when the bank will want to keep the agreement. If the manager is indifferent the bank is indifferent.

Lemma 3. If the solution for p_2 is interior for both scenarios then an extra dollar earned in the first period (an increase in y_1) leads to a bigger increase in the value of the renegotiation, $V_2^R(y_1, E, r^0)$, than in the value of keeping the agreement, $V_2^K(y_1, D)$.

The intuition behind the lemma is relatively simple. If the initial agreement is kept the bank holds the risky debt. For the manager the renegotiation is equivalent to refinancing the debt with another creditor. If the manager refinances, the risk that was initially held by the original lender is transferred to a new lender. The manager has to compensate the new lender for this risk, and the price increases with the amount of risky debt. By earning an extra dollar in the first period the manager decreases the amount of the risky debt, and, therefore, significantly reduces the price he must pay.

Note. Outside of the "interior" interval each function is either zero (for very low realization of y_1) or a linear with unit derivative (for very high realization of y_1).

Note 2. Assumptions A2 and A3 allow us to concentrate only the interior region and the region where either function is zero.

Corollary. If there exists $y_h \in (0, y^H)$: $V_2^R(y_h, E, r^0) = V_2^K(y_h, D)$ then

- 1) It is unique, and,
- 2) For $y_1 > y_h$ the manager's outside option is higher than his inside option and he will choose to renegotiate. For $y_1 < y_h$ the manager's outside option is lower than his inside option and the manager will choose to keep the initial agreement. However, in this case the bank will choose to renegotiate if the covenant is violated.

Note. If there is no such y_h we may define $y_h = y^H$ if $V_2^R(y_1, E, r^0) < V_2^K(y_1, D)$ for all $y_h \in (0, y^H)$, and $y_h = 0$ if $V_2^R(y_1, E, r^0) > V_2^K(y_1, D)$ for all $y_h \in (0, y^H)$.

FIGURE 4: Illustration for lemma 3

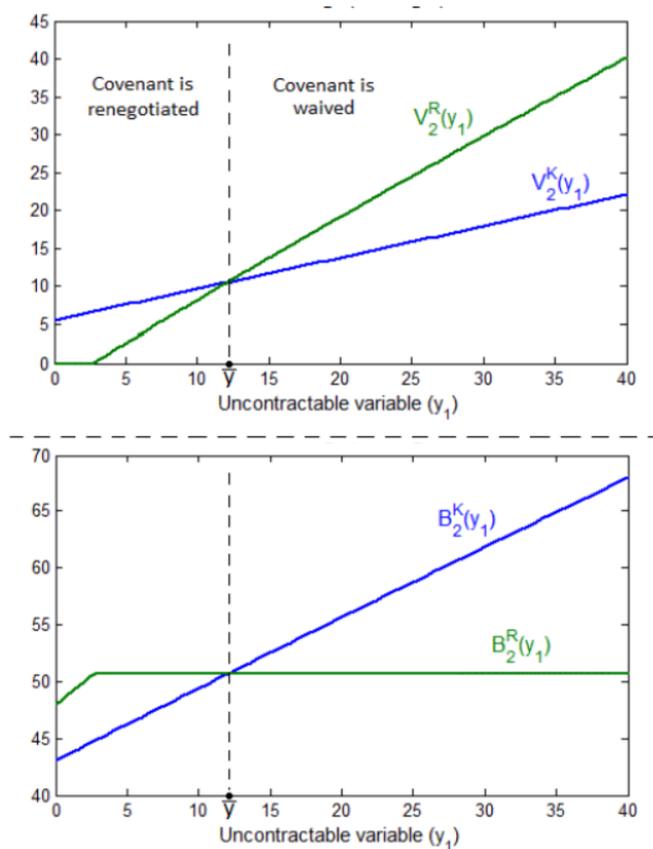


Figure 4a shows manager's continuation values of keeping the agreement or renegotiating it as functions of the uncontractable variable y_1 . If the realization of the variable y_1 is high (higher than the threshold level y), it is appealing to the outside investors and the manager will be able to refinance his project at a rate lower than his original contract. For that reason, the bank will

be able to adjust current contract, so, the renegotiation will result in a favorable (for the manager) result. If, on the other hand, the realization of y_1 is low (lower than \bar{y}) the manager's outside option is low and he prefers to stick to the agreement signed initially.

Figure 4b shows bank's continuation values of keeping the agreement or renegotiating it as functions of the uncontractable variable y_1 . The bank, on the other hand, will insist on a renegotiation if manager's outside option is low and the covenant is violated.

We can see that the violation of the covenant matters only if the realization of the uncontractable state is low. If this realization it is high the manager himself will choose to renegotiate the deal.

Lemma 4. The formula for the manager's continuation value is

$$V_1^{contr}(y_1, D, P_{contr}, E, r^0) = \begin{cases} P_{contr} \cdot V_2^R(y_1, E, r^0) + (1 - P_{contr}) \cdot V_2^K(y_1, D) & \text{if } y_1 \leq \bar{y} \\ P_{contr} \cdot V_2^R(y_1, E, r^0) & \text{if } y_1 > \bar{y} \end{cases}$$

The manager may affect the distribution of y_1 by choosing a costly effort e_1 . After the contract is signed, the manager treats contract parameters as exogenous values and picks effort e_1 to maximize

$$\max_{e_1} \left\{ \int_0^{y^H} V_1^{contr}(y_1, D, P_{contr}, E, r^0) \cdot g_1(y_1, e_1) dy_1 - C_1(e_1) \right\}$$

the optimal effort needs to satisfy the equation

$$\int_0^{y^H} V_1^{contr}(y_1, D, P_{contr}, E, r^0) \cdot \frac{\partial g_1(y_1, e_1)}{\partial e_1} dy_1 = C_1'(e_1)$$

Optimal effort depends on all the contract parameters and the exogenous parameters: $e_1^* = e_1^*(D, P_{contr}, E, r^0)$. The most important is to understand the connection between the covenant, P_{contr} , and the effort e_1^* . The tight covenant creates incentives for the manager because the outside option punishes the manager only for bad realizations of the variable y_1 .

The incentives increase the effort. Lemma 4 below formalizes the connection. The idea behind lemma 4 is that the accessibility of the outside option, generally, should increase the effort as it gives the incentives to the manager to show higher productivity specifically in the first period. However, there are two caveats. First is that for the region $y_1 > y$ the manager gets his outside option without any covenants involved. And second, for $y_1 < y$ manager's outside option is zero and it does not provide incentives if this region is too big.

Lemma 5. If

$$\int_{y_l}^{y_h} \left(\frac{\partial V_2^R(y_1, R, r^O)}{\partial y_1} - \frac{\partial V_2^K(y_1, D)}{\partial y_1} \right) \cdot \left[\int \frac{\partial g(z, e_1)}{\partial e_1} dz \right] dy_1 >$$

$$> \int_0^{y_l} \left(\frac{\partial V_2^K(y_1, D)}{\partial y_1} \right) \cdot \left[\int \frac{\partial g(z, e_1)}{\partial e_1} dz \right] dy_1$$

then

$$\frac{\partial e_1^*}{\partial P_{contr}} > 0$$

The proof is in the appendix (available upon request)

The term $\frac{\partial V_2^R(y_1, R, r^O)}{\partial y_1} - \frac{\partial V_2^K(y_1, D)}{\partial y_1}$ integrated over range $[y_l, y_h]$ represents the positive incentive effect of the covenant tightening. The difference between the derivatives in the "renegotiate" and the "keep" scenarios represents the capability of the covenants to create incentives. The term $\int \frac{\partial g(z, e_1)}{\partial e_1} dz$ is a positive term that reflects the fact that the distributions $G(\cdot | e_1)$ are F.O.S.D. ordered. The term $\frac{\partial V_2^K(y_1, D)}{\partial y_1}$ integrated over $[0, y_l]$ represents the loss in the incentives that strict covenants bring if manager's outside option is zero over a very big interval.

Corollary. If $y_l = 0$ (outside option exists even for very low realizations of y_1) and $D^{eq} < y^S$ (guarantees that the inside option is positive for $y_1=0$), tight covenant increases effort. If, on the other hand, $y > y^H$ (outside option is very bad) tight covenants decrease effort.

More general: there is a value $O = O(D)$ such that if $(1 + r^O) \cdot E < O(D)$ tight covenant increases incentives and effort, and if $(1 + r^O) \cdot E > O^*(D)$ tight covenant decreases optimal effort.

The covenant lowers manager's utility by giving the control to the bank. However, those control rights may be exercised only if the realization of y_1 is low. Therefore, the covenant creates an effective punishment for the low effort, since if the effort is low the probability of low y_1 is higher. In a favorable situation, when $(1 + r^0) \cdot E < O(D)$, tight covenant leads to the increase in the manager's effort. When the manager proposes a contract to the bank that contains a covenant he would like to commit ex-ante to the low payoff in case of low effort, and the channel of this commitment is the debt covenant.

3.5 The main theorem and discussion

As discussed before, that ex-post bank's control lowers manager's effort in the continuation of the project. If manager's outside option is zero the bank that has all bargaining power can effectively "liquidate" the firm by setting a very high debt at the renegotiation, making manager's effort at a lower bound and getting all proceeds from the project. There is a lower bound of the probability of success in the second period, and manager's effort is not needed, so, the bank can effectively replace the manager. This is inefficient from the social point of view. Assumption T7 implies that ex-post the bank wants it. If the bank can increase the final payment, it will.

If the manager has an opportunity to refinance the "liquidation" is impossible. But if the outside option is less than what the manager gets from the initial agreement the bank will still increase the face value of the debt and the manager's effort in the second period will be lower. This is a negative effect of the debt covenant. The theorem shows the effects that are balanced in the equilibrium. It is important to notice that tighter covenant allows the bank to decrease the initial interest rate (or, more specifically, the face value of the debt D). However, the covenant increases the risk of the liquidation, and because the parties behave as if they are risk-averse about the second period project, the covenant will not be used if it does not increase the effort e_1 .

We can formulate manager's problem as maximizing social value given the bank's constraint. To state the theorem, we need some notation. $W_2(y^1, D^*, P_{contr}^*, E, r^0)$ is the expected social value of the second period where the expectation is taken just after the availability of the information y_1 .

$$W_2(y^1, D^*, P_{contr}^*, E, r^0) = \begin{cases} (1 - P_{contr}) \cdot W_2^{keep}(y_1, D) + P_{contr} \cdot W_2^{renew}(y_1, E, r^0) & \text{if } y_1 \leq \bar{y} \\ W_2^{renew}(y_1, E, r^0) & \text{if } y_1 > \bar{y} \end{cases}$$

W_2^{keep} and W_2^{renew} are social values of the second project in both scenarios: $W_2^{keep}(y_1, D) = p_2^K(y_1, D) \cdot y^S - C_2(p_2^K)$; $W_2^{renew}(y_1, E, r^0) = p_2^R(y_1, E, r^0)y^S - C_2(p_2^R)$; p_2^K and p_2^R are equilibrium probabilities of success conditional on the outcome.

Theorem 1. Let (D^*, P_{contr}^*) be such that

- 1) the bank at least breaks even: $B(D^*, P_{contr}^*, E, r^0) \geq I$, and
- 2) for any other values $(\tilde{D}, \tilde{P}_{contr})$ such that $B(\tilde{D}, \tilde{P}_{contr}, E, r^0) \geq I$, the manager gets less: $V(D^*, P_{contr}^*, E, r^0) \geq V(\tilde{D}, \tilde{P}_{contr}, E, r^0)$, and
- 3) $V_2^K(0, D) > V_2^R(0)$, and
- 4) $P_{contr}^* \in (0, \overline{P}_{contr})$

then

- 1) $B(D^*, P_{contr}^*, E, r^0) = I$, and
- 2) the following equation is satisfied

$$\begin{aligned} & \frac{\partial e_1}{\partial P_{contr}} \left(\int_0^{y^H} y_1 \frac{\partial g_1}{\partial e_1} dy_1 - \frac{\partial C_1(e_1)}{\partial e_1} \right) + \\ & + \frac{\partial e_1}{\partial P_{contr}} \left(\int_0^{y^h} W_2(y^1, D^*, P_{contr}^*, E, r^0) \frac{\partial g_1}{\partial e_1} dy_1 \right) + \\ & + (1 - P_{contr}^*) \cdot \left(\frac{\partial B}{\partial P_{contr}} / \frac{\partial B}{\partial D} \right) \cdot \int_0^{\bar{y}} \left(- \frac{\partial W_2^{keep}(y_1, D)}{\partial D} \right) \cdot g_1(y_1, e_1) dy_1 = \\ & = \int_0^{\bar{y}} [W_2^{keep}(y_1, D^*) - W_2^{renew}(y_1, E, r^0)] g_1(y_1, e_1) dy_1 \end{aligned}$$

The term $\frac{\partial e_1}{\partial P_{contr}}$ captures the main role of covenants in this model: incentives effect to increase the manager's effort. For this effect to be positive, the outside option should be reasonably good as indicated in lemma 5. The term $\left(\int_0^{y^H} y_1 \frac{\partial g_1}{\partial e_1} dy_1 - \frac{\partial C_1(e_1)}{\partial e_1} \right)$ is the marginal effect of the increase in effort on the first period outcome. This term is positive if $e_1 < e_1^*$. If the effort is

already bigger than the optimal, the covenant should not be used and the term is negative. The term $\frac{\partial e_1}{\partial P_{contr}} \left(\int_0^{y^H} y_1 \frac{\partial g_1}{\partial e_1} dy_1 - \frac{\partial c_1(e_1)}{\partial e_1} \right)$ represents the marginal effect of tightening debt covenant on the first period outcome. We can call it "the direct incentive effect". The part $\left(\int_0^{y^h} W_2(y^1, D^*, P_{contr}^*, E, r^0) \frac{\partial g_1}{\partial e_1} \partial y_1 \right)$ is positive and captures the effect of the decrease in the debt overhang problem in the second period. Higher earnings in the first period increase manager's part of the final output and decrease the debt payment. This will increase the effort in the second period and the efficiency. We can call the second term of the equation "indirect incentives effect".

The tight covenant gives the bank a chance to collect more from the firm in certain cases. If the manager suggests a tighter covenant in the initial credit agreement, he can make the face value of the debt, D , lower. The third term of the equation above captures this effect. The product of $(1 - P_{contr})$ and $\left(\int_0^{\bar{y}} g_1(y_1, e_1) dy_1 \right)$ is the probability that the decrease in D will affect the final outcome (if the covenant is not violated or if the realization of y_1 is high enough the initial face value of the debt, D , does not matter for the final outcome). The term $\left(\frac{\partial B}{\partial P_{contr}} / \frac{\partial B}{\partial D} \right)$ is the size of the debt decrease due to P_{contr} increase. The term $\left(-\frac{\partial W_2^{keep}(y_1, D)}{\partial D} \right)$ is the social effect of debt decrease if the initial contract is kept. Overall the third term is the positive effect of the lower interest rate.

The last term is precisely the reason the probability of bank's control is not one in the model. Getting the control, the bank will be trying to push the manager towards his outside option. This will result in a higher debt at the start of the second period, and, therefore, lower effort and social surplus. The difference between social surpluses in "good" and "bad" scenarios is integrated over the interval where the covenant violation matters, i.e, $[0, \bar{y}]$. This effect can be called "moral hazard of the bank".

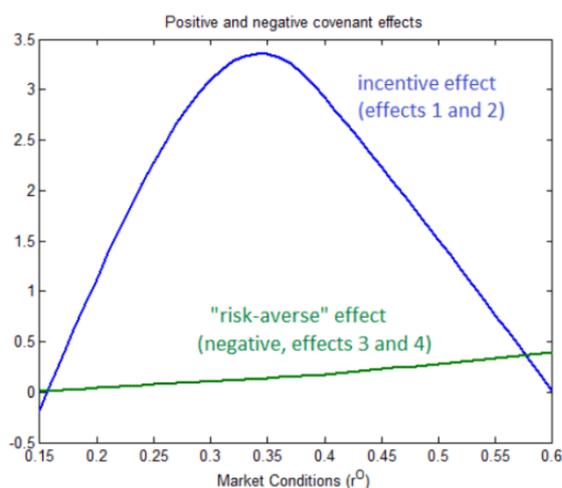
Terms 3 and 4 are alike as they directly affect the payment that the manager must pay in the second period. They work in the opposite directions as term 3 captures the effect of lower debt, term 4 captures the effect of higher bank's control. We can see, however, the effect of the term 4 is larger in the absolute value than the effect of the term 3. The reason for that is that the parties are risk-averse regarding the outcome of the second project. They would prefer a higher

payment with an average effort to a lottery that includes the possibility of a covenant violation which may result in the inefficient liquidation.

Conjecture. Consider a scenario with specified P_{contr} and D . Assume that the trade-off is only between the tightness of the covenant, P_{contr} , and the face value of the debt, D does not affect effort e_1 . The distribution of y_1 is fixed. In the second period the bank gets a lottery that can be described by the density of y_1 , the probability of the covenant violation and the final payment D . If the change of P_{contr} and D does not affect the distribution of y_1 , then the increase in P_{contr} (with the corresponding decrease of D) gives the bank a lottery that is a mean-preserving spread of the original lottery.¹⁰

Figure 5 illustrates all the effects described above.

FIGURE 5: Illustration for main theorem



The first function, the combined effects 1 and 2, shows that for reasonably good outside option the effect of the covenant tightening has a positive influence on the effort e_1 and the resulting efficiency. The effect is low for very good outside option as in this case the accessibility of the outside option is high without the covenant. The second function is the effect 4 minus effect 3. This function is always greater than zero and is increasing. This illustrates the risk-averse attitude, which is higher when the market conditions are worse. The worse market conditions are the more the expected punishment may be. The covenant will be positive whenever the first line is higher than the second.

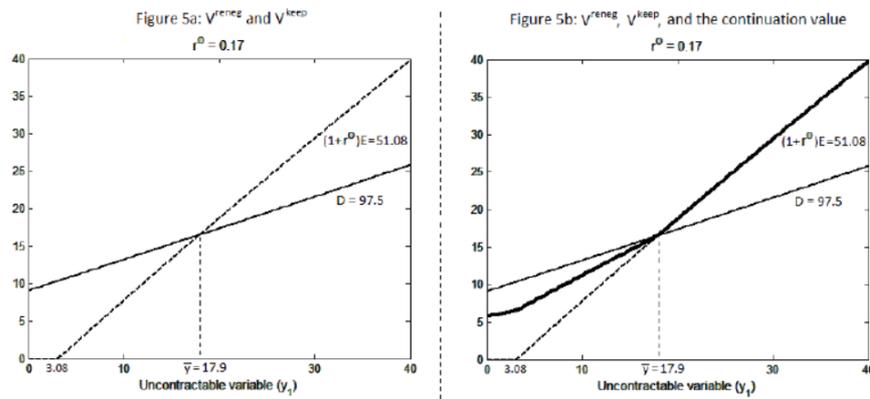
¹⁰ At the time of finishing the draft I did not have the general proof of the statement. This is true for the leading example in the paper.

Corollary. If the outside option is bad, i.e., $(1 + r^O) \cdot E > O(D)$, the optimal probability of bank's control is zero.

This is the consequence of the conjecture above and the statement in the third line of lemma 1.

The example of the interior solution is presented on the figure 6.

FIGURE 6: Illustration of covenant effects



On the figure 5a dashed line is the function $V_2^K(y_1, D)$ and solid line is the function $V_2^R(y_1, E, r^O)$. E and r^O are exogenous, picked so that $(1 + r^O) \cdot E = 51.08$. The optimal contract is $D = 97.5$ and $P_{contr} = 0.36$. If the probability of bank's control is zero the continuation value of the manager will be an upper envelope of the two functions. The "incentive package" that the manager has before exercising effort e_1 will consist of his outside option for high realizations of y_1 ($y_1 \geq y_h$) and the value of keeping the agreement for low realizations of y_1 . Since the value of keeping the agreement does not provide enough incentives, and the interval $[0, y_h]$ is large, the manager wants to pre-commit through the covenant. Tighter covenant further increases risk that the parties face and is not necessary.

With this we finish the discussion of the case with the pre-specified early payment amount, E , and turn my attention to a more general case with endogenous E .

4. Full model

If the efficient project is chosen, payment E is never realized in the equilibrium and only affects the outside option that the manager has during the renegotiation. If inefficient project exists, however, low payment E creates incentives for choosing the inefficient project. The role of the inefficient project is to restrict the endogenous choice of contractual repayment. By the project being "inefficient" I also imply that it gives lower expected return taking into account the second best solution (the solution under the specified environment) for the efficient project.

The initial contract should include the face value of the debt, D ; the agreement of the early repayment, E , and the probability that the bank will have control, P_{contr} . If the inefficient project is chosen the income is y_0 and the manager has a choice to return E in the first period or D in the second. Although contracts do not prohibit a situation where $D < E$ because of a very low outside option it'll never be optimal to design such a contract. We can assume that after the realization of y_0 in the first period the manager returns the debt E and keeps the rest.

If the manager picks the efficient project he expects to get $V(D, E, P_{contr}, r^0)$. The contract should be design so that the efficient project is always chosen.

Manager maximizes

$$\begin{aligned} \max_{D, E, P_{contr}} & V(D, E, P_{contr}, r^0) \\ s. t. & B(D, E, P_{contr}, r^0) \geq I \\ & V(D, E, P_{contr}, r^0) \geq y_0 - E \end{aligned}$$

It is easy to show that bank's participation constrain is always binding. The second constraint, however, may or may not be binding. We will call solution $(D^*(r^0), E^*(r^0), P_{contr}^*(r^0))$ unconstrained if $V(D^*(r^0), E^*(r^0), P_{contr}^*(r^0)) > y_0 - E^*(r^0)$.

The algorithm that solves the problem is as follows.

1) Solve the "Unconstrained Problem":

$$\begin{aligned} \max_{D,E,P_{contr}} V(D, E, P_{contr}, r^0) \\ \text{s. t. } B(D, E, P_{contr}, r^0) \geq I \end{aligned}$$

Let the solution to this problem be $(D^{UC}(r^0), E^{UC}(r^0), P_{contr}^{UC}(r^0))$

2) Check the constraint: if $V(D, E, P_{contr}, r^0) > y_0 - E$ then $D^{UC}, E^{UC}, P_{contr}^{UC}$ is the solution. Otherwise constraint is binding. Then we need to solve the following problem:

$$\begin{aligned} \max_{D,E,P_{contr}} V(D, E, P_{contr}, r^0) \\ \text{s. t. } B(D, E, P_{contr}, r^0) \geq I \\ V(D, E, P_{contr}, r^0) = y_0 - E \end{aligned}$$

which is equivalent to

$$\begin{aligned} \min_{D,E,P_{contr}} E \\ \text{s. t. } B(D, E, P_{contr}, r^0) \geq I \\ V(D, E, P_{contr}, r^0) = y_0 - E \end{aligned}$$

Lemma 6. If for both r_1^0 and r_2^0 are the solutions that are unconstrained then $D^*(r_1^0) = D^*(r_2^0)$ and $P_{contr}^*(r_1^0) = P_{contr}^*(r_2^0)$. Moreover, $(1 + r_1^0) \cdot E^*(r_1^0) = (1 + r_2^0) \cdot E^*(r_2^0)$.

If the manager picked the efficient project, payment E is never realized in the equilibrium. E affects manager's outside option which is also defined by $(1 + r^0)$. If the inefficient project does not exist at all, any changes in r^0 will be immediately compensated by the endogenous choice of E . Only the "competition" with the inefficient project will restrict the lowest value of E . If the constraint is not binding the two problems are identical, and, same as in the case of the absence of the inefficient project, any changes in r^0 will be compensated by changes in E .

As r^0 increases, corresponding E decrease and the "slack" of the constraint becomes less. We can formulate the theorem that captures the dependence of the solution type on the market conditions.

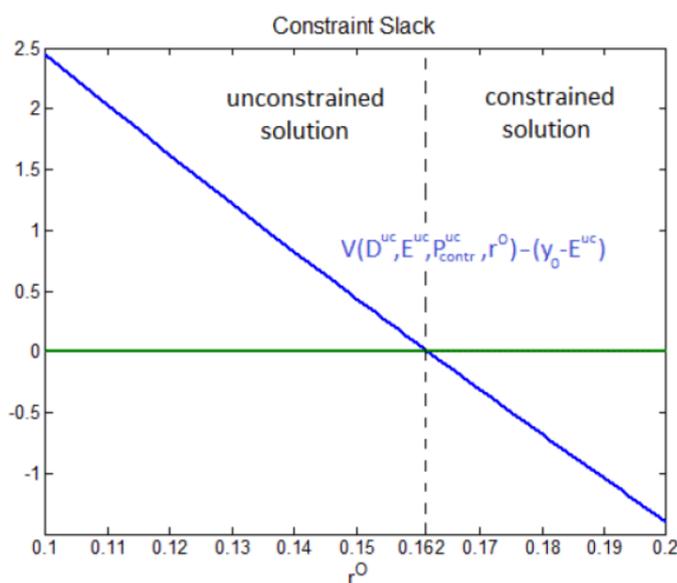
Theorem 2. Let V^{UNC} be the manager's value function at the unconstrained solution. It does not depend on r^0 and it implies some endogenous value of $(1 + r^0)E$. If $V^{UNC} > y_0 - E$ (the

constraint at $r^0 = 0$ is not binding) and $y_0 > V^{UNC}$, then there exists a value \bar{r}^0 such that if $r^0 < \bar{r}^0$ the solution is unconstrained, for $r^0 > \bar{r}^0$ the solution is constrained.

The proof of the theorem is in the appendix.

Figure 7 illustrates the change in the slack of the incentive constraint as r^0 changes.

FIGURE 7: illustration for theorem 2



$(D^{uc}, E^{uc}, P_{contr}^{uc})$ are the solutions to the unconstrained problem and functions of the exogenous parameter r^0 . As r^0 increases the resulting E decreases and the inefficient project becomes increasingly attractive to the manager. At some point the contract with low E becomes infeasible as the manager would choose the inefficient project instead. The equilibrium where manager chooses the inefficient project exists for some values of y_0 but I am not considering them since they are not interesting. In the leading numerical example in this paper the value V is around 20 (slightly less for very bad outside option) and the possible value of the inefficient project is 15 (as $y_0 = 65$ and $I = 50$).

In the next section, we will discuss the effect of changes in the loan market conditions on the solution. I will also look at the effect of a small change in y_0 which may be interpreted as a change of inefficiencies inside the firm or stealing opportunities.

5. Comparative statics

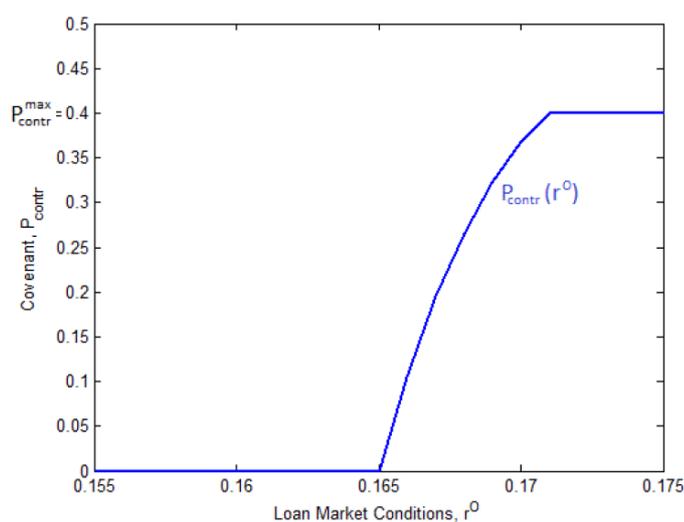
5.1 Loan market conditions

There is a value \tilde{r}^0 such that when the opportunity costs of funding for the banks, $r^0 > \tilde{r}^0$ financial markets are bad and the outside option does not re-introduce contingencies effectively. For the leading example the value of \tilde{r}^0 is around 0.60. The main comparative statics analysis will be concentrated on the region where $r^0 \ll \tilde{r}^0$. Although the numerical values are not informative, the goal of the paper is to analyze the effect of the introduction of the refinancing possibility, and the assumption that $r^0 \ll \tilde{r}^0$ implies that "good" firms have a high outside option. This is consistent with the way I interpret the "trip wire" story. The situation in which the waiver was granted corresponds to the high realization of y_1 in the model.

The analysis is most interesting around the value of r^0 at which the constraint becomes binding.

Figure 7 shows how covenant strictness depends on the market conditions, the graph of P_{contr} as a function of r^0

FIGURE 7: Loan market conditions and covenant



If $r^0 \leq 0.162$ nothing, including the covenant strictness, changes with the change of r^0 . Once $r^0 > 0.162$, the "effective" outside option, that is determined by $(1 + r^0) \cdot E^{eq}$ becomes worse and the analysis similar to the one in section 4 is applicable here.

Conjecture. In the area, close to $r^O \approx 0.162$ increase in the opportunity costs of funding for the outside banks leads to stricter debt covenants.

Intuition. At $r^O \leq 0.162$ the parties themselves contractually pick the best possible outside option.¹¹ There is a good chance that the manager will have a high outside option after the realization of y_1 . Once r^O increases over the threshold value 0.162, the outside option becomes worse. The probability that the outside option will be willingly picked decreases as the threshold y growth. This increases the manager's desire to commit to the outside option in case of bad performance and the equilibrium probability of bank's control will be higher.

Example. Figures 8 and 9 show the difference between good and bad conditions of the loan markets. For figures 8a and 9a $r^O = 1.6$, which is just below the threshold value 0.162, for figures 8b and 9b $r^O = 1.7$, which is just above.

FIGURE 8 (a,b): keep and renegotiate value functions

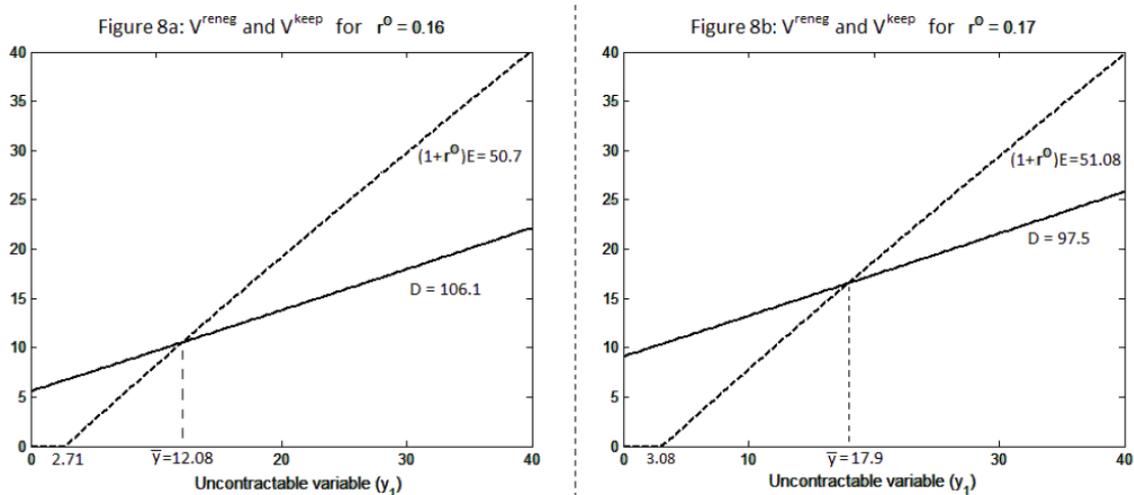
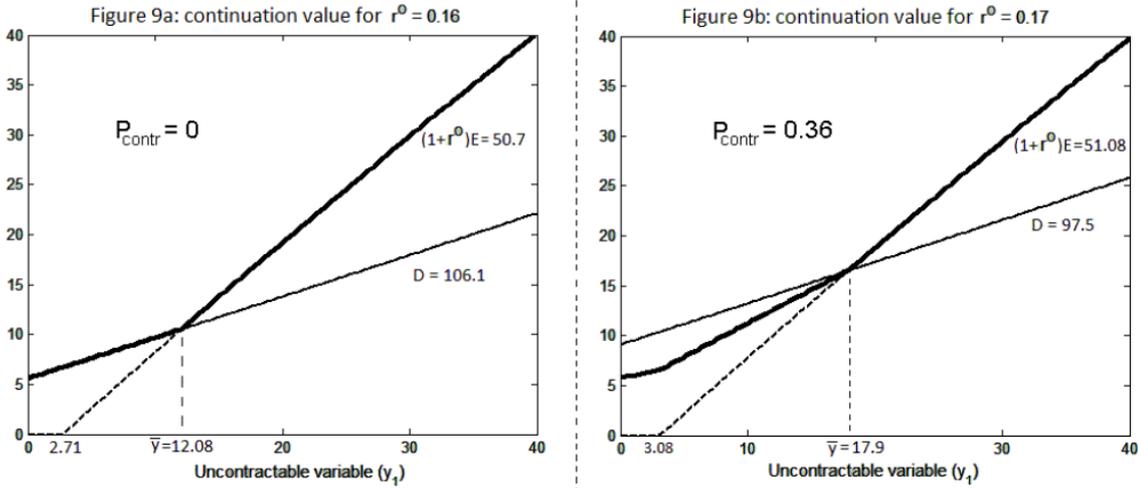


Figure 8a plots the value functions V^K and V^R at the equilibrium values D and E for $r^O=0.16$. The region where the manager gets his outside option, $[y, y^H]$, is large, so the chances high that there will be favorable renegotiation. In this situation, the equilibrium probability of bank's control is zero. For figure 8b, the region $[y, y^H]$, is smaller and the incentives provided by the possibility of a favorable (for the manager) renegotiation are smaller. The covenant in this

¹¹ The equilibrium may or may not include a positive probability of bank's control depending on the magnitude of bank's moral hazard problem.

situation provides more incentives as the manager is committed for an unfavorable renegotiation with some probability, which will result in a punishment. Figure 9 shows covenants and the continuation values for both cases.

FIGURE 9 (a,b): covenant and continuation value



5.2 Inefficiencies inside the firm

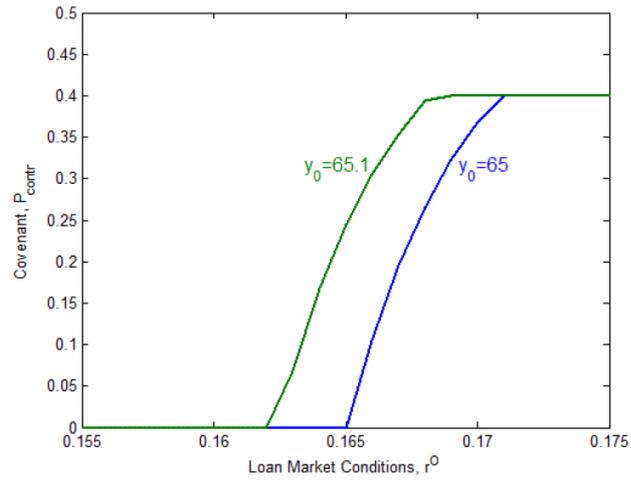
Debt covenants are observed to be tighter for intrinsically riskier firms. Equity holders tend to prefer riskier activities, and the higher the risk the more attractive the project may be for equity holders. The goal of the initial contract is to keep the manager, who acts on behalf of the equity holders, to stay away from the risky activities.

One way to analyze the effect of the increase in the risk of the inefficient activity would be to assume that instead of y_0 the inefficient project brings a lottery with the same expected value. This will increase the attractiveness of the project to the manager. An increase in y_0 is an equivalent way to analyze the risk increase. The inefficient project with higher y_0 becomes more attractive to the manager. Higher expected value is not important as the project is not chosen in the equilibrium.

The advantage of this approach is the simplicity.

Figure 10 illustrates the effect of a small increase of y_0 on the equilibrium probability of bank's control. The result is consistent with the observed empirical evidences that covenants are tighter for riskier firms.

FIGURE 10: inefficiencies inside the firm



6. Conclusion

In this paper, I explore the role of debt covenants in shaping future renegotiation. Specifically, I consider an incomplete contract between a bank and a manager with a possibility of refinancing with a competitive bank. The possibility of refinancing is not realized in the equilibrium but it determines the manager's outside option and the renegotiation outcome. First, I show that the outside option generally creates better incentives for the manager than the inside option. But since this is an "option" the manager will not choose it if it is worse than the inside option. I show that the outside option is worse than the inside option for low realizations of the uncontractable variable and better than the inside option for high realizations. The primary use of debt covenants in this model is to make the outside option accessible for low realizations of the uncontractable variable. This creates a punishment for the manager and creates ex-ante incentives to avoid the punishment.

Ex-post, however, debt covenants are detrimental because they create a risk that the parties would like to avoid ex-ante. If debt covenants are tight there is a risk that the bank will have very high bargaining power which might lead to the liquidation in certain states. The optimal covenant balances the ex-ante incentive effect and the ex-post inefficiencies.

Manager's outside option is determined by two components: 1) the loan market conditions that I assume are known before the contract is signed, and 2) the outcome of the uncontractable variable. After the loan market conditions are known parties foresee how the outside option will interact with actions taken by the manager and sign a contract that makes this interaction most efficient. The threshold that defines the area where the outside option is better than the inside option is determined by the loan market conditions. If the conditions are good the interval where the outside option is better than the inside option is bigger. This creates higher incentives for the manager and debt covenants might not be needed. Once market conditions worsen the interval where the manager voluntarily chooses the renegotiation decreases and the ex-ante commitment through debt covenants becomes more important. This leads to an interesting empirical prediction that covenants are tighter if conditions of refinancing are expected to worsen.

The model can be generalized in two directions. The first direction is that correlation between the uncontractable variable and the contractile variable may be taken into account. In the present version, only special case is analyzed where the correlation is zero. Higher correlation

creates a more complete contracting framework. It might be interesting to see whether covenants are tighter or looser if contracts are more complete and compare the results with Sridhar and Magee (1997). The second restrictive assumption is that the opportunity costs for the lender at the time of signing the agreement is not correlated with the opportunity costs for lenders at the renegotiation stage. The opportunity cost for the lender at signing the initial agreement is normalized to zero. The model can be generalized to take into account this correlation. This will strengthen the interpretation of the comparative statics properties with respect to the loan market conditions. When we talk about worsening market conditions we have to stress that the future market conditions will be worse, while at the time of signing the agreement nothing changes. If we take this correlation into account this restriction is relaxed.

The model allows us to look at debt covenant design from a different perspective by suggesting that the default option does not have to be too bad for the manager. If market conditions are good and the health of the firm is sound despite the violation, the manager should be able to find alternative capital. Then he will be in a good position at the renegotiation with the lender. In most cases the refinancing has significant deadweight loss comparing to the renegotiation and rarely is the outcome. Common logic and the model suggest that if the borrower stayed with the lender it does not mean he did not have access to alternative funds. That could be his credible outside option which he did not pick because of the inefficiency. Even in case study it might be very hard to understand whether the firm that violated a covenant had an outside option. The paper suggests a framework where this question may be attempted to be tested empirically.

References

Aghion, P. and Bolton P., 1992. An Incomplete Contract Approach to Financial Contracting, *Review of Economic Studies*, vol. 59.

Berlin, M. and Mester, L., 1992. Debt Covenants and Renegotiation, *Journal of Financial Intermediation*, vol. 2.

Bradley M. and Roberts M., 2014. The Structure and Pricing of Corporate Debt Covenants, *Quarterly Journal of Finance*, vol. 5(2).

Demerjian P. 2015. Uncertainty and Debt Covenants, working paper.

Dichev, I. and Skinner, D., 2002. Large sample evidence on the debt covenant hypothesis, *Journal of Accounting Research*, vol. 40.

Diamond, D., and He, Z., 2014. A Theory of Debt maturity: The Long and Short of Debt Overhang, *Journal of Finance*, vol. 69(2), pp. 719-762.

Garleanu, N. and Zwiebel, K., 2008. Design and Renegotiation of Debt Covenants, *Review of Financial Studies*, Vol. 22(2), pp. 749-781.

Grossman, S. and Hart, O., 1986. The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration, *Journal of Political Economy*, vol. 94.

Jensen, M. and Meckling, J., 1976. Theory of the Firm: Managerial Behavior, Agency Costs and Ownership, *Journal of Financial Economics*, vol. 3(4), pp. 305-360.

Hart, O. and Moore, J., 1988. Incomplete Contracts and Renegotiation, *Econometrica*, vol. 56.

Hart, O. and Moore, J., 1990. Property Rights and the Nature of the Firm, *Journal of Political Economy*, vol. 98.

Hart, O. and Moore J., 1998. Default and Renegotiation: A Dynamic Model of Debt, *Quarterly Journal of Economics*, vol. 113.

Huberman, G. and Kahn, C., 1988. Limited Contract Enforcement and Strategic Renegotiation, *The American Economic Review*, vol. 78.

Maskin, E. and Tirole, J., 1999. Unforeseen Contingencies and Incomplete Contracts, *The Review of Economic Studies*, vol. 66.

Moyen, N. 2007. How Big is the Debt Overhang Problem? *Journal of Economic Dynamics and Control*, vol. 31.

Murfin, Justin, 2012. The Supply-Side Determinants of Loan Contract Strictness, *The Journal of Finance*, vol. 67

Myers, S., 1977. Determinants of corporate borrowing, *Journal of Financial Economics*, vol. 5.

Roberts, M. and Chava C., 2008. How does Financing Impact Investment? The Role of Debt Covenants, *Journal of Finance*, vol. 63.

Roberts, M. and Sufi, A., 2009. Control Rights and Capital Structure: An Empirical Investigation, *Journal of Finance*.

Roberts, M. and Sufi, A., 2009. Financial Contracting: A Survey of Empirical Research and Future Directions, *Annual Reviews of Financial Economics*.

Roberts, M. and Sufi A., 2009. Renegotiation of Financial Contracts: Evidence from Private Credit Agreements, *Journal of Financial Economics*

Segal, I., 1999. Complexity and Renegotiation: A Foundation for Incomplete Contracts, *The Review of Economic Studies*, vol. 66.

Sridhar, S. and Magee, R., 1997. Financial Contracts, Opportunism, and Disclosure Management. *Review of Accounting Studies*, vol. 1.

Stiglitz, J. and Weiss, A., 1983. Incentive Effects of Terminations: Applications to the Credit and Labor Markets, *American Economic Review*, vol. 73.