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Real Business Cycles in The Model with Two-Person Household and Home Production

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Abstract

In the U.S. economy hours and productivity are negatively correlated, and volatility of hours is two times higher than volatility of productivity. In the standard one shock RBC model hours are positively correlated with productivity, and hours are two times less volatile than productivity. This paper is an attempt to replicate the co-movement of hours and productivity observed in the post-war U.S. data using one shock model. I explore the real business cycles in the model with two-person household and home production. The model economy has a representative household of two agents. Agents allocate their time among leisure, work on the market and home production. There is a fixed cost of working on the market, and agents may choose not to work. The fluctuations in the model are driven by aggregate technology shock. I calibrate the model to U.S. data, solve and simulate it. I find that in the model hours are 2 times more volatile than productivity, and that hours and productivity are negatively correlated. The model replicates well the co-movement of hours and productivity observed in the U.S. data.

Keywords: Computable General Equilibrium, Business Cycles, Home Production, Labor Supply

JEL Classification: C68, E32, E24, J22

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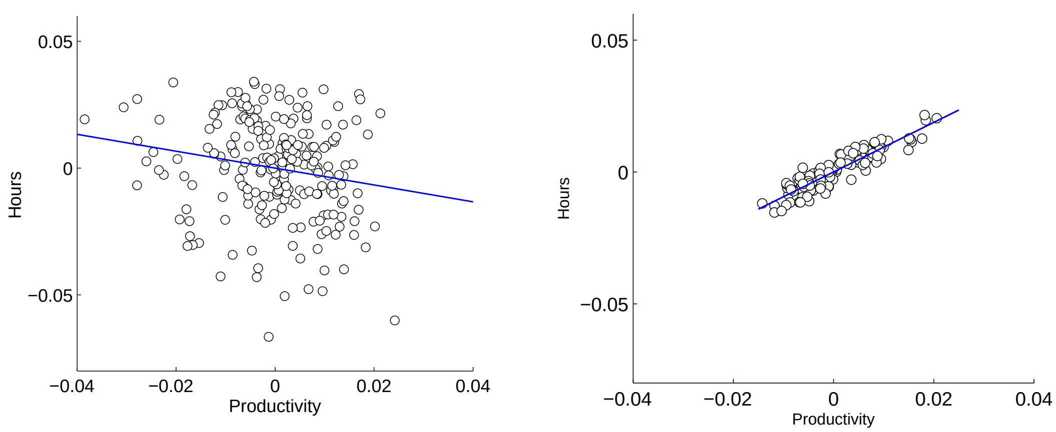
1 Introduction

The standard neoclassical model does a good job in accounting for many business cycle properties of the U.S. economy, but in some areas it fails. One of the most important failures is the inability to account for the co-movement of hours and productivity along the business cycle. The standard model predicts that changes in the productivity are the main determinant of the changes in hours worked. In the standard model the correlation of hours and productivity is positive, and hours are 2 times less volatile than productivity. But the U.S. economy exhibits exactly opposite behavior: the correlation of hours and productivity is negative[1], and hours are almost 2 times more volatile than productivity. These differences between the predictions of the standard RBC model and the U.S. data can be seen at Figure 1

Figure 1: Hours and Productivity



A: U.S. economy B: Standard neoclassical RBC model

All variables are logged and detrended with Hodrick-Prescott filter (Hodrick and Prescott, 1997).

In an attempt to solve the problem, Hansen (1985) proposed a model with indivisible labor in the representative agent setup. His model increased the volatility of output and hours, but did not improve on other accounts. Cho and Rogerson (1988) suggested that two-person household is a useful assumption: it allows to study the labor market participation decision. The possibility to exit the labor market increased the volatility of hours, and also changed the behavior of hours with respect to productivity. Important part of the literature departed from one-shock tradition, and introduced additional shocks that would influence the agent's decision on hours and diminish the impact of productivity. Another type of shock could be the shock to government spending

[1] The correlation coefficient of the aggregate hours and aggregate productivity in the U.S. (both variables are logged and detrended) is -0.18. The correlation coefficient is statistically significant at 1% level.

(Christiano and Eichenbaum, 1992), the shock to the alternative sector, like home production (Benhabib et al., 1991; Greenwood and Hercowitz, 1991), or idiosyncratic shock to individual production opportunities of the agents (Díaz-Giménez, 1997). New shocks decreased the correlation of hours and productivity, but did not change its sign.

Robert Lucas defined business cycle as a co-movement of economic series:

> Let me begin to sharpen the discussion by reviewing the main qualitative features of economic time series which we call "the business cycle". (...) These movements do not exhibit uniformity of either period or amplitude. (...) Those regularities which are observed are in the co-movements among different aggregative time series. [2]

Co-movement of hours and productivity in the post-war U.S. data can be characterized by two observations: (1) correlation of hours and productivity is negative; (2) hours are two times more volatile than productivity. The goal of this paper is to replicate this co-movement in the one shock model.

The standard model focuses on the representative household of one person, who has no option of exiting the labor market, and has only two time-use options: work or leisure. This paper considers the model with two-person representative household, where one of the household members can choose to exit the labor market, and both agents have three options of time use: working on the market, working at home and leisure. Another goal of this paper is to evaluate the impact of introduction of two-person households or home production separately.

Although it is difficult to measure the volume of home-produced goods, the importance of home production sector can be easily evaluated just by looking at the data on factors of production. According to the Bureau of Labor Statistics American Time Use Survey (ATUS) for 2007 (see Horrigan and Herz, 2005), average American spends 25% of his discretionary time working on the market, and a respectable 13% working at home, and this is excluding time spent on child care. Several papers (f.e. Gomme and Rupert, 2007) stress the importance of considering the home production sector in calibration, even if the model itself does not consider home production explicitly.

Home production is especially important for married couples. Around 60% of working-age population in United States are married, so two-person household is not a strong assumption. Married couples usually pool income and make decisions together, and that affects their labor

[2] Lucas, (1977).

market decisions even if one does not consider children. Guler et al. (2009), for example, explore joint labor decisions in the context of search theory. Greenwood and Guner (2008) explore long-run technology growth in the model with couples and home production to explain recent changes in labor market participation of men and women. There are reasons to believe that introduction of two-person household can also change the cyclical properties of the standard model: adjustment to the changes in productivity in the multi-agent household happens not only through changes in time allocation of one agent, but also through change in allocation of hours worked among household members.

This paper studies the cyclical properties of the model economy with two sectors, market and home production, and with a representative household of two individuals. Two agents pool income and make all the economic decisions jointly. The introduction of the home sector introduces new opportunity for use of hours. There is also a fixed cost of working in the market, and the agents may choose to exit the labor force. I calibrate the model to the U.S. economy. I solve the model using value function iterations. I find that in this model volatility of hours relative to output is 6 times higher than in RBC model and 2 times higher than in the data. Volatility of hours is twice the volatility of productivity, and correlation of hours and productivity is negative, as in the U.S. data.

Since it is important to distinguish between the effects of introducing home production and two-person households, I start with a home production model in representative one-person household (Section 2), continue with a two-person household model without home production (Section 3) and conclude with a full-featured model of two-person household, home production and fixed cost of working (Section 4).

2 Home Production Model (H)

In this section I study the properties of the home production model, that is referred to as *model H* in the rest of the paper. It's main differences from the Benhabib, Rogerson and Wright (1991) model are the absence of uncertainty in home production and the absence of capital in the home production technology. These assumptions are needed to simplify computations in the final stage of the paper, and in this chapter I will also evaluate their effect over the performance of the model.

American Time Use Survey defines home production as "household activities are those done by persons to maintain their households. These include housework; cooking; lawn and garden care; pet care; vehicle maintenance and repair, home maintenance, repair, decoration, and renovation; and household management and organizational activities (such as filling out paperwork, balancing a checkbook, or planning a party". Note that home production in this interpretation does not include time spent on child care.

2.1 The model

There is one infinitely-living representative agent in the model. At each period of time representative agent maximizes the discounted sum of future utilities, and utility is derived from consumption of the composite good c_t and leisure l_t. Here and thereafter subscripts h and m refer to home and market production.

$$U_t = \sum_t^{\infty} \beta^t u_t \qquad \beta \in (0,1) \tag{1}$$

$$u_t = \frac{({c_t}^{\sigma} {l_t}^{1-\sigma})^{1-\gamma}}{1-\gamma} \tag{2}$$

Composite good is a CES combination of market-produced consumption $c_{m,t}$ and home-produced good $c_{h,t}$. Note that $\frac{1}{1-e}$ in this case is the elasticity of substitution between home and market-produced goods, and a is the weight of market consumption.

$$c_t = (a{c_{m,t}}^e + (1-a){c_{h,t}}^e)^{1/e}, \qquad a \in (0,1), e \in (-\infty, 1) \tag{3}$$

Agent is endowed with T units of time each period, and he allocates time among leisure l_t, hours worked in the market $h_{m,t}$ and hours worked at home $h_{h,t}$:

$$T = l_t + h_{m,t} + h_{h,t} \tag{4}$$

Market goods are produced with Cobb-Douglas production function that is subject to technology shock z_t:

$$Y_{m,t} = e^{z_t} {K_t}^{\alpha} {L_{,t}}^{1-\alpha}, \qquad \alpha \in (0,1) \tag{5}$$

Technology shock follows the following autocorrelation process:

$$z_t = \eta z_{t-1} + \epsilon_t, \qquad \epsilon \sim N(0, \rho^2) \tag{6}$$

Market good is consumed or invested. Note that investment good is produced only in the market:

$$Y_{m,t} = c_{m,t} + i_t \tag{7}$$

An agent also has access to home production that is produced and consumed within household. To simplify further computations I assume that only labor ($h_{h,t}$) is used in home production, and that home production sector is not subject to any shock. Labor displays diminishing returns.

$$c_{h,t} = A{h_{h,t}}^{1-\alpha_1}, \qquad \alpha_1 \in (0,1) \tag{8}$$

At period $t = 0$ the household is endowed by k_0 units of capital. We assume $k_0 \in (0, \infty)$. Capital depreciates at the rate δ. Capital moves according to the following law:

$$k_{t+1} = k_t(1-\delta) + i_t \tag{9}$$

2.2 Definition of equilibrium

Given initial conditions $k_0 > 0$ and z_0, a competitive equilibrium for a vector of technology shocks $\{z_t\}_{t=0}^{\infty}$ consists of household policy, $\{c_{m,t}(w,r,k),\ h_{m,t}(w,r,k),\ h_{h,t}(w,r,k),\ i_t(w,r,k)\}_{t=0}^{\infty}$, firm's policy $\{K_t(w,r,z),\ L_t(w,r,z)\}_{t=0}^{\infty}$, and a vector of prices, $\{r_t, w_t\}_{t=0}^{\infty}$, such that:

(i) Given the vector of factor prices $\{r_t, w_t\}$ and the state vector $\{k_t\}$, the household policy solves household maximization problem:

$$\max E[\sum_{t=0}^{\infty} \beta^t (\frac{({c_t}^{\sigma} {l_t}^{1-\sigma})^{1-\gamma}}{1-\gamma})] \tag{10}$$

subject to:

$$c_t = (a{c_{m,t}}^e + (1-a){c_{h,t}}^e)^{1/e} \tag{11}$$

$$c_{m,t} + i_t = w_t h_{m,t} + r_t k_t \tag{12}$$

$$c_{h,t} = A{h_{h,t}}^{1-\alpha_1} \tag{13}$$

$$T = l_t + h_{m,t} + h_{h,t} \tag{14}$$

$$k_{t+1} = k_t(1-\delta) + i_t; \tag{15}$$

(ii) Given the vector of factor prices $\{r_t, w_t\}$ and productivity shock $\{z_t\}$, aggregate quantities $\{K_t, L_t\}$ solve the firm's maximization problem:

$$\max\{Y_{m,t} - w_t L_t - r_t K_t\} \tag{16}$$

subject to:

$$Y_{m,t} = e^{z_t} {K_t}^{\alpha} {L_{,t}}^{1-\alpha} \tag{17}$$

(iii) The markets clear:

$$K_t = k_t \tag{18}$$

$$L_t = h_{m,t} \tag{19}$$

$$c_{m,t} + i_t = Y_{m,t} \tag{20}$$

2.3 Calibration

Since the focus of this paper is the behavior of hours, main calibration targets are hours worked in the market and at home. According to American Time Use Survey (2008), in 2008 an average American spent 3.73 hours on work or work-related activities and 1.73 on household activities a day. Two parameters, σ, the relative importance of consumption, and a, the weight of market good in the composite good are calibrated to match average hours worked in the market and average hours worked at home. Parameter A, total factor productivity of home production, is calibrated to match the share of home production in GDP - lower bound estimate of 20%. Since there is no capital in the model, I adopt the parameter α_1 from Benhabib et al. (1991), where it was set to match the share of home production investment in total capital stock. The rest of the parameters are calibrated to conventional values.

2.4 Discussion and results

The introduction of home production changes the standard model in the following ways:

- Third option of time use appears, now in case of the bad shock an agent can move hours not only into leisure, but into home production. As a result, the change in the market hours is not necessarily connected with the change in total consumption.

- In standard model adjustment (and volatility) of market hours translated directly into leisure; as an agent prefers low volatility of leisure over time, it translated into low market hours volatility. Now you can change both market and home hours, while leisure remains unchanged. That explains the increase in volatility of hours.

- The second sector of the economy - home production - is very different from the market sector, because in every given moment of time an agent has total control of the amount produced and labor productivity in this sector. There is no capital, that is given every period of time in the market sector, there is no uncertainty brought by a technology shock. This is also the main difference from Benhabib et al. (1991) and Greenwood and Hercowitz (1991).

All the business-cycle properties of the H model are documented in Appendix A. Here are he most important differences between the home production model and standard RBC model:

(a) relative volatility of hours is almost 3 *times higher*

Home production is an additional option to use time, so now, if the bad shock happens, an agent might decrease hours worked in the market, and then distribute them among leisure and hours worked at home. Now the changes in hours worked in the market do not translate directly into changes in consumption or leisure, and this allows an agent to adjust market hours on bigger margins.

(b) relative volatility of investment decreases

In the standard model adjustment of hours spent on the market is limited, because adjusting the hours also means changing leisure and consumption, which is something the agent wants to avoid. Therefore, more adjustment was made on the capital/investment side. Now, with the possibility of adjusting hours without changing leisure and consumption a lot, more adjustment is made on the hours side, less adjustment on the investment.

(c) the volatility of output is almost doubled

The driving force behind the increased volatility of output is high volatility of hours. The agent does not have very high incentives to smooth out the market output, because now it can be compensated by home-produced goods.

(d) relative volatility of productivity is reduced by factor of 6

Now an agent can escape the bad shock in the market by shifting hours into home production, maintaining the productivity of market labor more stable. An agent wants the productivity (in utility terms) of hours spent working on the market and to home to be equal, and since the productivity at home is almost constant (there are diminishing returns in home production, but marginal productivity decreases very slowly compared to market production function), agent adjusts market hours in such a way that market productivity does not oscillate much. In H model home production does not depend on accumulated capital or on exogenous technology shock, hence it is a perfect vehicle of escape from the bad shock. That is why in H model volatility of productivity is even less than in other home production models. For the same reason (absence of shock and capital in home production) correlation of productivity with output is higher than in Benhabib, Rogerson and Wright (1991) - relative productivity is the most important factor for allocating labor between two sectors, but in our case relative productivity depends mostly on the market productivity. Perfect negative correlation of market productivity and home hours confirms it.

One of the goals of this section was to evaluate the impact of omission of capital and technology shock from the home production technology. Table 3 documents the main differences between home production model featured in this paper and the one used by Benhabib, Rogerson and Wright (1991). It focuses on the statistics of interest, i.e. those that the neoclassical growth model fails to replicate.

Table 1: Home production model: Comparisons with Benhabib, Rogerson and Wright, 1991

	$corr(w, h_m)$	$\sigma(h_m)/\sigma(w)$	$\sigma(y)$	$\sigma(h_m)/\sigma(y)$	$\sigma(i)/\sigma(y)$
U.S. economy	–0.18	1.82	2.03	0.95	2.35
RBC *	0.92	0.52	1.19	0.39	2.97
BRW**	0.49	1.92	1.71	0.75	2.73
H***	0.91	8.32	2.31	0.90	2.77

* Here and in the rest of the paper I use quarterly U.S. data from 1948:Q1 till 2004:Q2 (226 observations for each series). Source of data: Haver Analytics.
**Model economy used in Benhabib, Rogerson and Wright (1991)
***Model economy with home production

- Output volatility. While both BRW model and our H model do better than standard RBC model accounting for output volatility, they are both equally away from the U.S. economy.

- Relative volatility of investment Both models produce similar statistic and are an improvement over the standard model. The omission of home capital in our home production model clearly did not affect the behavior of market investment.

- Relative volatility of hours worked. Our home production model is an improvement both over standard RBC model and the model used by Benhabib, Rogerson and Wright (1991).

- Correlation of output and productivity. BRW model overperformes the standard RBC model. This result in BRW model is driven by the presence of home-production specific technology shock. Now the relative productivity of the market sector is important for allocation of labor and the output volume, therefore market productivity has limited impact on output (and hours). Since we omit the home-production specific technology shock, H model delivers result similar to RBC model.

- Correlation of hours and productivity. Again BRW model is an improvement over RBC model, for the reasons described in the previous point. Model H again performs as the standard RBC.

To sum up, the omission of capital from home production technology did not have a significant effect on the model performance, while the omission of home-production specific technology shock had important impact on the behavior of productivity. On the other hand, while results of BRW model are better, they are still far from the values observed in the U.S. economy.

The introduction of home production in otherwise standard model improved its performance, increasing volatility of hours and output and decreasing relative volatility of investment. Still it does not explain the behavior of productivity and hours observed in the data.

3 Model with Two-Person Households with (2PF) and without (2P) fixed cost of working

This section analyzes the business cycle properties of the model with a representative two-person household. I assume that all the decisions in the household are Pareto optimal, and therefore we can assume that they are centralized. There is no bargaining, and the main reasons are the absence of outside options and the fact that divorces do not happen at the business cycle frequency. Two agents have similar preferences and differ only in productivity. I study 2 setups of the model: with the fixed cost of working (denoted 2PF) and without it (2P).

3.1 The Model

The household maximizes the infinite discounted stream of household utility, and at each point in time utility of the household is the weighted sum of utilities of two agents. lets call the agents a man (subscript 1) and a woman (subscript 2). Note that each agent gains utility from shared consumption and individual leisure. In this case shared consumption does not represent any economies of scale for the household, and the results would be the same if utility was defined over $\frac{c}{2}$.

$$u_t = \mu \frac{({c_t}^{\sigma} {l_{1,t}}^{1-\sigma})^{1-\gamma}}{1-\gamma} + (1-\mu) \frac{({c_t}^{\sigma} {l_{2,t}}^{1-\sigma})^{1-\gamma}}{1-\gamma} - \tau I(t), \qquad \mu \in (0,1) \tag{21}$$

There is a fixed cost of working τ. It can be interpreted as any cost associated with working, like extra expenditure on child care, commuting cost or simply the discomfort of leaving home and spending time at a workplace. $I(t)$ is the indicator function that takes a value of 1 if both agents are working on the market in period t.

It's not restrictive to assume that agent 2 is less productive than agent 1:

$$H_t = h_{1,t} + \nu h_{2,t}, \qquad \nu \in (0,1) \tag{22}$$

where H_t are total productive hours worked on the market. Each agent has a total time endowment T that is distributed between leisure $l_{i,t}$ and hours worked on the market $h_{i,t}$:

$$T = l_{i,t} + h_{i,t} \tag{23}$$

At period $t = 0$ the household is endowed by k_0 units of capital. We assume $k_0 \in (0, \infty)$. Capital depreciates at the rate δ. Capital moves according to the following law:

$$k_{t+1} = k_t(1-\delta) + i_t \tag{24}$$

3.2 Definition of equilibrium

Given initial conditions $k_0 > 0$ and z_0, a competitive equilibrium for a vector of technology shocks $\{z_t\}_{t=0}^{\infty}$ consists of household policy, $\{c_t(w,r,k),\ h_{1,t}(w,r,k),\ h_{2,t}(w,r,k),\ i_t(w,r,k)\}_{t=0}^{\infty}$, firm's policy $\{K_t(w,r,z),\ L_t(w,r,z)\}_{t=0}^{\infty}$, and a vector of prices, $\{r_t, w_t\}_{t=0}^{\infty}$, such that:

(i) Given the vector of factor prices $\{r_t, w_t\}$ and the state k_t, the household policy solves household maximization problem:

$$\max E[\sum_{t=0}^{\infty} \beta^t (\mu \frac{({c_t}^{\sigma} {l_{1,t}}^{1-\sigma})^{1-\gamma}}{1-\gamma} + (1-\mu) \frac{({c_t}^{\sigma} {l_{2,t}}^{1-\sigma})^{1-\gamma}}{1-\gamma} - \tau I(t))] \tag{25}$$

where

$$I(t) = \begin{cases} 1 & \text{if } h_{1,t}h_{2,t} \neq 0 \\ 0 & \text{if } h_{1,t}h_{2,t} = 0 \end{cases}$$

subject to:

$$c_t + i_t = w_t h_{1,t} + \nu w_t h_{2,t} + r_t k_t \tag{26}$$

$$T = l_{1,t} + h_{1,t} \tag{27}$$

$$T = l_{2,t} + h_{2,t} \tag{28}$$

$$k_{t+1} = k_t(1-\delta) + i_t; \tag{29}$$

(ii) Given the vector of factor prices $\{r_t, w_t\}$ and shocks $\{z_t\}$, aggregate quantities K_t, L_t solve the firm's maximization problem:

$$\max\{Y_t - w_t L_t - r_t K_t\} \tag{30}$$

subject to:

$$Y_t = e^{z_t} {K_t}^{\alpha} {L_{,t}}^{1-\alpha} \tag{31}$$

(iii) The markets clear:

$$K_t = k_t \tag{32}$$

$$L_t = h_{1,t} + \nu h_{2,t} \tag{33}$$

$$c_t + i_t = Y_t \tag{34}$$

3.3 Calibration

Parameters β (discount factor), δ (depreciation rate) are assigned their conventional values, as in Cooley and Prescott (1995). There are, however, four parameters left to calibrate. Relative productivity of labor, ν, was calibrated to match the average wage difference between men and women. As average wage of a woman is only 70% of the wage of a man, ν is set to 0.7.

For the version of the model without the fixed cost the rest of the parameters were obtained from the equilibrium conditions for the non-stochastic version of the model. The two targets were average hours worked by men and average hours worked by women. Two parameters, utility weight parameter μ, leisure parameter σ, were calibrated to match those targets.

As for the version with the fixed cost of working, parameters could not be determined by solving for equilibrium conditions of non-stochastic version of the economy, because in non-stochastic version the market job participation is 100%. Hence the fixed cost parameter τ could not be calibrated, as it does not affect the outcome in non-stochastic equilibrium. So I had to find the rest of parameters by guess. I had three parameters to calibrate: utility weight parameter μ, weight of consumption in utility parameter σ and the fixed cost τ. Three targets I chose were average hours worked by men and women and average job market participation rate of women (since a man always works in the model). The results of calibration are summarized in Table 2.

Table 2: Values for 2PF Model Parameters

	Parameter	Value	Calibration Method
Relative productivity	ν	0.70	from data
Weight of consumption	σ	0.46	by search
Relative utility weight	μ	0.50	by search
Fixed cost of work	τ	0.38	by search
Discount factor	β	0.99	conventional value
Depreciation rate	δ	0.025	conventional value
Autocorrelation of the shock	ρ	0.95	conventional value
St.dev. of the shock	η	0.007	conventional value

3.4 Business Cycle Properties of 2P and 2PF models

As expected, 2P is not much different from RBC. Small differences that can be observed lie well within the range of one standard deviation of corresponding statistic, and may be considered insignificant. Complete set of business cycle statistics for model 2P (without fixed cost) is in Appendix A.

In contrast, the model with the fixed cost of labor force participation produces some interesting results. In bad times a woman chooses to exit the labor force, because staying there is costly and productivity is low. Her exit causes the raise in productivity, and man tries to compensate it by working more hours. But the compensation is only partial, and the total hours fall, while productivity stays high. So in bad times hours fall (by more significant amount than in standard RBC model), while productivity goes up. This explains why in the 2PF model productivity is countercyclical. Note that since the fall in hours is accompanied by the rise in productivity, correlation of hours and productivity is negative. Correlation coefficient is larger in absolute value than the one we see in the data.

Table 3: Two-person household model: Comparisons

	$corr(w, h_m)$	$\sigma(h_m)/\sigma(w)$	$\sigma(y)$	$\sigma(h_m)/\sigma(y)$	$\sigma(i)/\sigma(y)$
U.S. economy	–0.18	1.82	2.03	0.95	2.35
RBC *	0.92	0.52	1.19	0.35	2.97
2P*	0.97	0.29	1.23	0.23	2.40
2PF**	–0.99	1.79	11.04	2.25	2.85

* Model economy with two-person household (without fixed cost)
**Model economy with two-person household and fixed cost of working

As you can see, the model economy with the fixed cost is very volatile. The source of volatility is hours. Hours are volatile by construction - the model was calibrated to deliver the 60% average rate of job market participation of women, and for this model it means that in 40% of times a woman stays at home (since in each period of time participation of women may be only 100% or 0%). Even though a man compensates the exit of a woman from the labor market by increase in his hours, that's not enough.

Absolute volatility od investment also rises to accommodate for the drastic change in hours. But on relative scale investment is less volatile than it is in standard RBC model.

There are two main drawbacks of 2PF model: (1) the economy is excessively volatile, (2) productivity is countercyclical. The main advantage of the 2PF model (over standard RBC or home production model) is the negative correlation of hours and productivity.

Complete set of business cycle statistics for 2PF model can be found in Appendix A.

4 Model with Two-Person Households, Home Production and Fixed Cost of Working (H2PF)

In previous sections I investigated the effects of introducing home production or two-person household separately. In this section I calibrate solve and simulate a model that combines two-person household with home production. Agents face a fixed cost of labor market participation. At each period of time a household has 6 decisions to make: whether a woman should enter the labor market, how much time a man and a woman should spend working in the market and at home and how much to invest.

4.1 The Model

H2PF model has two economic agents: an infinitely living household of two people, that supplies factors of production, and the firm, that uses these factors to produce the consumption and investment good. Lets describe the model formally.

4.1.1 Preferences

There are two individuals in the household, lets call them a man and a woman. The household is infinitely living and maximizes the infinite discounted stream of utilities.

$$U_t = \sum_{t}^{\infty} \beta^t u_t \qquad \beta \in (0,1) \tag{35}$$

The utility of each person $i = 1, 2$ (where subscript 1 stands for a man and subscript 2 stands for a woman) in each point in time depends on the level of shared composite consumption c_t and individual leisure $l_{i,t}$:

$$u_{i,t} = \frac{({c_t}^{\sigma} {l_{i,t}}^{1-\sigma})^{1-\gamma}}{1-\gamma} \tag{36}$$

The household utility is the weighted sum of utilities of a man and a woman:

$$u_t = \mu \cdot u_{1,t} + (1-\mu) \cdot u_{2,t} - \tau \cdot I(1,t) \cdot I(2,t), \qquad \mu \in (0,1) \tag{37}$$

There is a fixed cost of working τ. It can be interpreted as any cost associated with working, like extra expenditure on child care, commuting cost or simply the discomfort of leaving home and spending time at a workplace. $I(i,t)$ is the indicator function that takes a value of 1 if an agent i is working on the market in period t. Note that utility is derived from consumption of the composite good c_t. It is composed of the consumption good produced on the market $c_{m,t}$ and home-made consumption good $c_{h,t}$ (as in Benhabib, Rogerson and Wright, 1991).

$$c_t = (a{c_{m,t}}^e + (1-a){c_{h,t}}^e)^{1/e}, \qquad a \in (0,1), e \in (-\infty, 1) \tag{38}$$

4.1.2 Home Production

Home production function only uses labor. I omit capital and home-production specific technology shock to simplify calculations. In home production, both a man and woman are equally productive, and labor demonstrates decreasing returns to scale:

$$c_{h,t} = Ah_{1,h,t}{}^{1-\alpha_1} + Ah_{2,h,t}{}^{1-\alpha_1}, \qquad \alpha_1 \in (0,1) \tag{39}$$

where $h_{1,h,t}$ and $h_{2,h,t}$ are hours spent on homework by a man and a woman respectively.

4.1.3 Market Technology

Production function for goods sold on the market is a standard Cobb-Douglas production function, that is subject to technology shock z_t:

$$Y_{m,t} = e^{z_t} K_t{}^{\alpha} L_{m,t}{}^{1-\alpha} \tag{40}$$

Technology shock follows the following autocorrelation process:

$$z_t = \eta z_{t-1} + \epsilon_t, \qquad \epsilon \sim N(0, \rho^2) \tag{41}$$

$L_{m,t}$ is aggregate labor:

$$L_{m,t} = h_{1,m,t} + \nu h_{2,m,t}^{w}, \qquad \nu \in (0,\infty) \tag{42}$$

Parameter ν reflects relative productivity of women in the market production. Only market good may be used for investment; it can also be consumed:

$$Y_{m,t} = c_{m,t} + i_t \tag{43}$$

4.1.4 Endowments

Total time endowment T of agent i is distributed among leisure l_t^i, hours worked in the market sector $h_{i,m,t}$ and hours worked in home production $h_{i,h,t}$.

$$T = l_{i,t} + h_{i,m,t} + h_{i,h,t} \tag{44}$$

At period $t = 0$ the household is endowed by k_0 units of capital. We assume $k_0 \in (0,\infty)$. Capital depreciates at the rate δ. Capital moves according to the following law:

$$k_{t+1} = k_t(1-\delta) + i_t \tag{45}$$

4.2 Definition of equilibrium

Given initial conditions $k_0 > 0$ and z_0, a competitive equilibrium for a vector of technology shocks $\{z_t\}_{t=0}^{\infty}$ consists of household policy, $\{c_{m,t}(w,r,k)$, $h_{1,m,t}(w,r,k)$, $h_{2,m,t}(w,r,k)$, $h_{1,h,t}(w,r,k)$, $h_{2,h,t}(w,r,k)$, $i_t(w,r,k)\}_{t=0}^{\infty}$, firm's policy $\{K_t(w,r,z),\ L_t(w,r,z)\}_{t=0}^{\infty}$, and a vector of prices, $\{r_t, w_t\}_{t=0}^{\infty}$, such that:

(i) Given the vector of factor prices $\{r_t, w_t\}$ and the state k_t, the household policy solves household maximization problem:

$$\max E[\sum_{t=0}^{\infty} \beta^t (\mu \frac{(c_t{}^{\sigma} l_{1,t}{}^{1-\sigma})^{1-\gamma}}{1-\gamma} + (1-\mu)\frac{(c_t{}^{\sigma} l_{2,t}{}^{1-\sigma})^{1-\gamma}}{1-\gamma} - \tau I(t))] \tag{46}$$

where

$$I(t) = \begin{cases} 1 & \text{if } h_{1,m,t}h_{2,m,t} \neq 0 \\ 0 & \text{if } h_{1,m,t}h_{2,m,t} = 0 \end{cases}$$

subject to:

$$c_t = (ac_{m,t}{}^{e} + (1-a)c_{h,t}{}^{e})^{1/e} \tag{47}$$

$$c_{h,t} = Ah_{1,h,t}{}^{1-\alpha_1} + Ah_{2,h,t}{}^{1-\alpha_1} \tag{48}$$

$$c_{m,t} + i_t = w_t h_{1,m,t} + \nu w_t h_{2,m,t} + r_t k_t \tag{49}$$

$$T = l_{1,t} + h_{1,m,t} + h_{1,h,t} \tag{50}$$

$$T = l_{2,t} + h_{2,m,t} + h_{2,h,t} \tag{51}$$

$$k_{t+1} = k_t(1-\delta) + i_t; \tag{52}$$

(ii) Given the vector of factor prices $\{r_t, w_t\}$ and shocks $\{z_t\}$, aggregate quantities K_t, L_t solve the firm's maximization problem:

$$\max\{Y_{m,t} - w_t L_t - r_t K_t\} \tag{53}$$

subject to:

$$Y_{m,t} = e^{z_t} K_t{}^{\alpha} L_{,t}{}^{1-\alpha} \tag{54}$$

(iii) The markets clear:

$$K_t = k_t \tag{55}$$

$$L_t = h_{1,m,t} + \nu h_{2,m,t} \tag{56}$$

$$c_{m,t} + i_t = Y_t \tag{57}$$

4.3 Calibration

As before, a set of parameters (β, δ, ρ, η) are assigned their conventional values, as in Cooley and Prescott (1995). I also assume that utility curvature parameter γ is equal to 1, or, in other words, utility is logarithmic, and that e equals to 1, meaning that there is an infinite elasticity of substitution between home-produced and market goods. I need the last two assumptions, because they decrease substantially the computation time needed to solve the economy. Logarithmic utility is computed 10 times faster than utility with other values of γ. Assumption of infinite elasticity of substitution between two kinds of consumption allows to derive the hours worked at home as a function of hours worked in the market. As in Section 3, relative productivity of labor, ν, was calibrated to match the average wage difference between men and women. As average wage of a woman is only 70% of the wage of a man, ν is set to 0.7.

Table 4: Values for H2PF Model Parameters

	Parameter	Value	Calibration Method
Relative productivity	ν	0.70	from data
TFP of home production	A	0.46	by search
Weight of market good in consumption	a	0.15	by search
Curvature of home production function	α_1	0.50	by search
Weight of consumption	σ	0.29	by search
Relative utility weight	μ	0.72	by search
Fixed cost of work	τ	0.12	by search
Discount factor	β	0.99	conventional value
Capital share	α	0.36	conventional value
Depreciation rate	δ	0.025	conventional value
Autocorrelation of the shock	ρ	0.95	conventional value
St.dev. of the shock	η	0.007	conventional value

There are 6 parameters left to calibrate: home production parameters A and α_1, relative weight of man's utility μ, fixed cost parameter τ, weight of consumption in utility parameter σ and weight of marked-produced goods in composite consumption a. There are also 6 targets: average job market participation rate for women (60%), average hours worked by women in the market and in home production, average hours worked by men in the market and in home production (according to the ATUS data) and the lower-bound estimate for the volume of home production as a share of GDP (20%). The values of the parameters were calibrated to those targets by search. The calibrated values of parameters are in Table 4.

Note that the calibrated value of the fixed cost is very small - it is less than 1% of average utility level, and is comparable to the loss of 6.67% of composite consumption, or the loss of 40 hours of man's time quarterly (around 26 minutes per day, or 3 hours a week). Hence the fixed cost of working in this case is comparable to commuting cost an average American faces every day (its 25 minutes a day according to Census).

4.4 Results

In this section I discuss the business cycle properties of the model with home production, two-person households and fixed cost of working. I compare them with the properties of the data. I also analyze the behavior of home production sector separately.

4.4.1 Comparisons between the United States and the H2PF Economy: Amplitude of Fluctuations

Table 5: The Amplitude of the Fluctuations

	$\sigma(x)$			$\sigma(x)/\sigma(y)$		
Series	Data	RBC	H2PF*	Data	RBC	H2PF*
Output	2.03	1.19 (0.09)	8.68 (0.28)	1	1	1
Market Consumption	0.88	0.50 (0.04)	1.34 (0.09)	0.43	0.42	0.15
Investment	4.78	3.53 (0.27)	39.99 (1.37)	2.35	2.97	4.61
Capital Stock	0.64	0.31 (0.04)	1.84 (0.15)	0.32	0.26	0.21
Market Hours	1.93	0.41 (0.03)	18.81 (1.21)	0.95	0.35	2.17
Productivity	1.06	0.79 (0.06)	10.30 (0.36)	0.52	0.66	1.19

*Model economy with two-person households, home production and fixed cost.

- H2PF economy is very volatile - its 4 time more volatile than the US economy. The main source of volatility is hours.
- Hours are 9 times more volatile than in the data.There are two reasons: possibility to substitute not only into leisure, but also into home production (intensive margin), and possibility of exiting the labor market to avoid paying the fixed cost (extensive margin).
- The adjustments on the extensive margin are very significant. If in the data the participation rate changes only 1-3%, in our economy a woman exiting the job market means a 30-40% drop in market hours. So the model economy is too volatile by construction.
- High volatility in hours is the cause of high volatility in investment. In case of a bad shock hours drop (because a woman exits the labor market), and the interest rate falls even more (than in RBC model in case of a bad shock).

- Productivity is also more volatile, but if in the standard RBC model it goes down in case of bad shock, here it goes up, because of a significant drop in hours. And although there is also a sharp drop in investment at the time, it does not have immediate effect on productivity.

- Market consumption is also more volatile than in the data or in the RBC model. The main reason for it is the possibility to offset it by home consumption. In relative terms, however, market consumption in H2PF is less volatile. It is directly connected to the increased volatility of investment - now in case of bad shock there is a huge drop in investment, and relatively smaller one in consumption.

4.4.2 Comparisons between the United States and the H2PF Economy: The Correlations between the Variables

Table 6: Correlations

Correlations with Output			
Series	Data	RBC	H2PF*
Output	1	1	1
Market Consumption	0.77	0.97 (0.01)	0.54 (0.03)
Investment	0.74	0.99 (0.00)	0.99 (0.00)
Capital Stock	0.07	0.04 (0.03)	–0.09 (0.04)
Market Hours	0.86	0.98 (0.00)	0.98 (0.00)
Productivity	0.35	0.99 (0.00)	–0.96 (0.01)
Correlations with Productivity			
Market hours	–0.18	0.92 (0.01)	–0.99 (0.00)

*Model economy with two-person households, home production and fixed cost.

- Market consumption is less correlated with output than in the data. Again it can be explained by the fact that in case of bad shock women exit the job market, investment falls sharply (in response to the fall in hours and interest rate), and consumption does not fall as much as it would fall without women exiting the job force.

- Market hours are strongly procyclical, as they are in the US economy and in RBC model, although in the models they are slightly higher.

- Productivity in the H2PF model is countercyclical, and this result contradicts the data and differs from standard RBC conclusion. The negative correlation of productivity with output is the direct consequence of fixed cost. In the event of bad shock labor supply falls so drastically that productivity actually goes up, hence the countercyclicality.

Table 7: Home Production Sector in H2PF Model Economy: Volatility

Series	$\sigma(x)$	$\sigma(x)/\sigma(y)$
Total Consumption	1.24 (0.04)	0.14
Market Consumption	1.34 (0.09)	0.15
Home Consumption	6.25 (0.24)	0.72
Total Hours	1.93 (0.14)	0.22
Market Hours	18.81 (0.64)	2.17
Home Hours	24.56 (0.93)	2.82
Home Hours	24.56 (0.93)	2.82
Home Hours by men	5.70 (0.20)	0.66
Home Hours by women	35.58 (1.27)	4.10

4.4.3 Home Production Sector in the H2PF Model Economy

The statistics characterizing the volatility of fluctuations of the home production sector are in the Table 7.

- Total consumption is less volatile than market consumption or home consumption, because two sectors offset each other.
- The same is even more true about total hours: market and home almost perfectly offset each other.
- Home production sector is in general more volatile, as it is completely controlled by the household at each point in time (there is no uncertainty and no dynamic component like capital).
- Market hours are less volatile than the home hours, and this finding is in line with the data[3]. Lower volatility of home hours by women suggests that when women exit the labor force, they do not transfer all the time into home production.

The correlations of the home production sector variables with market output are in the Table 8.

- Home production sector is countercyclical. In case of a bad shock household transfers available hours into home production.

[3]According to the ATUS quarterly data for 2003:1-2008:4, volatility of hours spent in home production is 1.62 times higher than the volatility of hours worked on the market.

Table 8: Home Production Sector in H2PF Model Economy: Correlations

Correlations with Output	
Market Consumption	0.54 (0.03)
Home Consumption	–0.99 (0.00)
Market Hours	0.98 (0.00)
Home Hours	–0.99 (0.00)
Home hours by men	0.89 (0.02)
Home hours by women	–0.99 (0.00)

- There are important differences in labor market behavior of men and women. Due to the presence of fixed cost, in bad times a woman exits the labor force. A man supplies more hours on the market, but not enough to compensate a woman's exit, so the total market hours fall. As the result, productivity goes up. This explains why productivity, market hours by men and home hours by women go up in bad times (and, therefore, they are countercyclical), and why market hours by women and home hours by men are countercyclical.

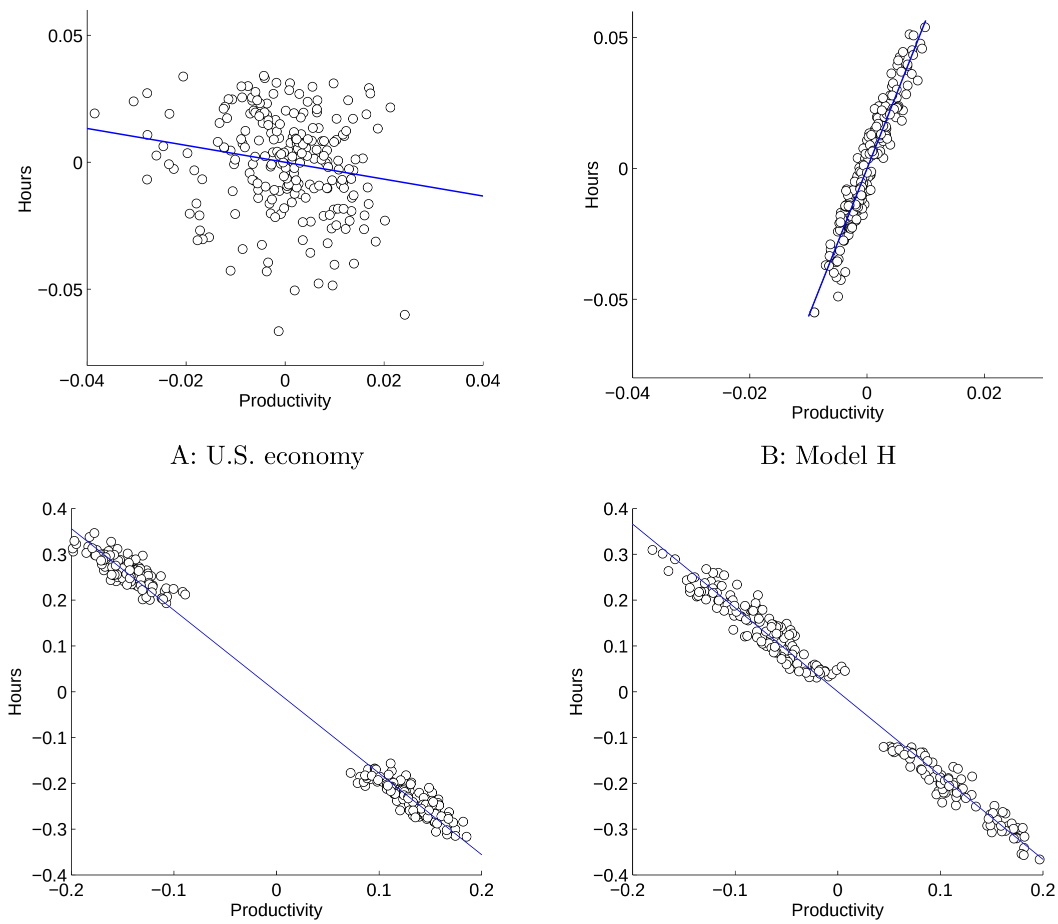
5 Concluding comments

In this paper I estimate the business-cycle properties of the model of two-person household with home production and fixed cost of labor. I find that the model describes very well the comovement of hours and productivity observed in the U.S. economy. In the model hours and productivity are negatively correlated, and hours are two times more volatile than productivity, like in the post-war U.S. data.

The performance of the model can be evaluated by looking at Figure 2. Introduction of home production only increases volatility of hours relative to the volatility of productivity (part B of Figure 2), but it does not change the positive correlation between the series. Introduction of two-person households with the fixed cost of working increases volatility of hours both relative to output (6 times versus the RBC model) and productivity (4 times versus the RBC model). Introduction of two-person household also reverses the correlation of hours and productivity from positive to negative (parts C and D of Figure 2). Full set of business cycle statistics can be found in Appendix A. The model with both home production and two-person households (H2PF model) describes the best the observed co-movement of hours and productivity.

The model is also successful in replicating the main business cycle properties of the home production sector, such as countercyclicality and high volatility relative to output.

Figure 2: Hours as a Function of Productivity



A: U.S. economy

B: Model H

C: Model 2PF

D: Model H2PF

All variables are logged and detrended with Hodrick-Prescott filter (Hodrick and Prescott, 1997).

Note that the scales of the graphs A and B differ from the scales of the graphs C and D. The reason for that is the incomparable volatility of hours and productivity in those models: the amplitude of fluctuations is higher in the models with two-person household (2P and H2Pf).

Still there are drawbacks. As a result of huge changes in job market participation rate, hours fluctuate too much relative to output (twice as much as in the data), but this excessive volatility stems from the way the model was calibrated. Productivity is negatively correlated with output, while in the data there is a small positive correlation.

In this model two-person households and fixed cost of labor market participation are crucial to explain the joint behavior of hours and productivity. A natural extension of this research would be to test the robustness of the results to the introduction of other households. Main result of the model, negative correlation between hours and productivity, is driven by the fact that a man can not fully compensate for the labor market exit of a woman. What if there were other agents, will the result hold? This question deserves further exploration.

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A Business Cycles in the U.S. economy and 5 model economies

Table 9 documents the business cycle properties of the post-war U.S. economy and 5 model economies. All variables are logged and detrended with Hodrick-Prescott filter before computing statistics. For each model economy 100 samples of similar length (226 periods) were simulated; the statistics in the table are average values.

- Data - Quarterly data on the U.S. economy from 1948:Q1 till 2004:Q2 (226 observations for each series). Source of data: Haver Analytics.
- RBC - The standard neoclassical business cycle model with conventional parameter values.
- H - A model with home production, described in Section 2 of this paper.
- 2P - A model with representative two-person household, but without fixed cost of working; described in Section 3 of this paper.
- 2PF - A model with representative two-person household, with fixed cost of working; described in Section 3 of this paper.
- H2PF - A model with representative two-person household, fixed cost of working and home production; described in Section 4 of this paper.

Table 9: Business Cycle Properties of the U.S. Economy and 5 Model Economies

	Data	RBC	H	2P	2PF	H2PF
Co-movement of market hours (h) and productivity ($w = y/h$)						
$corr(h, w)$	–0.18	0.92	0.91	0.97	–0.99	–0.99
$\sigma(h)/\sigma(w)$	1.82	0.52	8.32	0.29	1.79	1.82
Amplitude of fluctuations, relative to output						
Output	2.03	1.19	2.31	1.11	11.04	8.68
Consumption	0.43	0.42	0.53	0.64	0.51	0.15
Investment	2.35	2.97	2.77	2.40	2.85	4.61
Capital	0.32	0.26	0.24	0.21	0.09	0.21
Productivity	0.52	0.66	0.11	0.78	1.25	1.19
Total hours			0.25			0.22
Market hours	0.95	0.35	0.90	0.23	2.25	2.17
Home hours			1.37			2.82
Market hours	0.95	0.35	0.90	0.23	2.25	2.17
Market hours by men				0.19	1.29	1.05
Market hours by women				0.28	26.75	29.32
Home Hours			1.37			–0.99
Home hours by men						0.89
Home hours by women						–0.99
Correlations with output						
Consumption	0.77	0.97	0.93	0.99	0.98	0.54
Investment	0.74	0.99	0.96	0.99	0.99	0.99
Capital	0.07	0.04	0.11	0.07	–0.27	–0.09
Productivity	0.35	0.99	0.93	0.99	–0.98	–0.96
Total hours			0.76			
Market hours	0.86	0.98	0.99	0.96	0.99	0.98
Home hours			–0.93			–0.99
Market Hours	0.86	0.98	0.99	0.96	0.99	0.98
Market hours by men				0.96	-0.99	-0.99
Market hours by women				0.96	0.99	0.98
Home hours			–0.93			–0.99
Home hours by men						0.89
Home hours by women						–0.99
Correlations with Productivity						
Total hours			0.52			
Market hours	–0.18	0.92	0.91	0.97	-0.99	-0.99
Home hours			–1.00			0.99
Market hours	–0.18	0.92	0.91	0.97	-0.99	-0.99
Market hours by men					0.99	0.99
Market hours by women				–0.99	–0.99	
Home Hours			–1.00			–0.99
Home hours by men						0.89
Home hours by women						–0.99